TEACHING MATHEMATICAL PROBLEM SOLVING

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FOUNDER
MATH PLUS ACADEMY
9 → 0 + 9 = 9
18 → 1 + 8 = 9
27 → 2 + 7 = 9
36 → 3 + 6 = 9
45 → 4 + 5 = 9
54 → 5 + 4 = 9
63 → 6 + 3 = 9
72 → 7 + 2 = 9
81 → 8 + 1 = 9
90 → 9 + 0 = 9
Math is an ADVENTURE
– PAUL LOCKHART, MATHEMATICIAN’S LAMENT
PROBLEMS VS. EXERCISES

- Exercise - You know a way to solve the problem
- Problem - You do NOT know a direct way to solve the problem

Example: Sally has 24 cookies. If she shares equally among 3 people, how many will each person get?
JUSTIFYING PROBLEM SOLVING

1. Inquiry & problem solving are at the heart of mathematics and science

2. Provides opportunities to develop perseverance

3. Students who experience problem based learning perform BETTER on standardized tests.†

†Boaler, J (1998) Open and Closed Mathematics: Student Experiences and Understandings
YOU CAN’T AFFORD NOT TO DO THIS!
WITHOUT PROBLEM SOLVING...

• Collection of algorithms, formulas, and definitions to be memorized

• Reduces to just “answer-getting”

• Intrigue, discovery, and joy are removed
FROM PROCEDURAL FLUENCY TO PROBLEM SOLVING
SHIFTING PRACTICE

• Start with “rich” tasks
• Launch with curiosity
• Encourage productive struggle
• Orchestrate a productive classroom discussion
WHAT IS A RICH TASK?

• Low barrier to entry
• Fosters productive struggle, creativity
• Open to multiple strategies and representations
• Allows for making connections and conceptual understanding
• Generally done collaboratively
A farmer has a total of 13 chickens and cows. His wife notices that there are 42 legs. How many of each animal are on the farm?
PROBLEM-CENTERED CLASSROOMS

• Students often lack problem solving strategies beyond guess and check

• Teachers find it difficult to anticipate more than one way to solve a problem
TEACHING A PROBLEM SOLVING FRAMEWORK
PROBLEM SOLVING FRAMEWORK

JAMES TANTON

GEORGE POLYA

JOHN MASON
ENTRY

• What do I KNOW?
• What do I WANT?
• What can I introduce?
• Get started with easy wins
  • Start with an easy case
  • Add / remove a constraint
ATTACK

• Draw a Picture or Diagram

• Make a table or organized list

• Go to Extremes

• Experiment & Be Creative

• Seek Pattern & Structure

• Solve a Simpler Problem

• Wishful Thinking

• Guess, Check and Revise
REVIEW

- Is your solution answer reasonable?
- Compare to conjectures and estimates
- Share strategies and solutions
- Check for efficiency and generality
EXTEND

- Generalize the solution
- Use “What-If-Not” to explore similar problems
WHAT - IF - NOT

- List all attributes of the problem including “givens”
- Ask “what if not” for each attribute?
- Choose a new question to investigate
A farmer has a total of 13 chickens and cows. His wife notices that there are 42 legs. How many of each animal are on the farm?
PROBLEM SOLVING PROCESS

What do I KNOW?
What do I WANT?
What can I INTRODUCE?
Write the essentials of the problem in YOUR OWN WORDS

TRY A STRATEGY
Ask yourself:

DRAW A PICTURE OR DIAGRAM
MAKE AN ORGANIZED LIST
EXPERIMENT & BE CREATIVE
GO TO EXTREMES

ENTRY

ATTACK
A box of chocolates adorned the kitchen counter.

When Jake saw it, he ate 1/6 of the box.

Along came Joe and he ate 1/5 of what Jake left.

Then came Jill who ate 1/4 of what remained.

Later, Jeff ate 1/3 of the remaining chocolates.

I got there and I ate 1/2 of what Jeff left.

After I left, only 4 chocolates were uneaten.
Tina had 3 times as much money as Timmy. After Tina spent $60 and Timmy spent $10, they each had an equal amount of money left. How much money did Tina have at first?
How many stars in this row?
WOULD YOU RATHER...?

- **A**: 1/3 off the regular price
- **B**: 1/3 more at regular price

**Options:**
- Regular size
- Regular price
PROBLEM SOLVING IS A MEANS OF LEARNING MATH—NOT JUST A WAY OF APPLYING IT
GETTING THE MOST OUT OF PROBLEM SOLVING
DISCUSSIONS
(SMITH & STEIN 2011)

1. Anticipate student strategies
2. Monitor student work
3. Select students who should contribute
4. Sequence contributions (e.g. concrete to abstract)
5. Connect - help students see connections
CHALLENGES IN FACILITATING DISCUSSIONS

• Lack of teacher’s control
• Demands quick reactions
• Requires deep, interconnected knowledge of content, pedagogy, and students
PURPOSE OF THE 5 PRACTICES

• To make student-centered instruction more manageable by moderating the degree of improvisation required by the teachers and during a discussion.
A fourth-grade class needs five leaves each day to feed its 2 caterpillars. How many leaves would the students need each day for 12 caterpillars?

Use drawings, words, or numbers to show how you got your answer.

Do this problem in as many ways as you can, both correct and incorrect.
## Monitoring

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<th>Strategy</th>
<th>Who / What</th>
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QUESTIONS

• How might carefully selecting and sequencing students’ responses affect the quality of the discussion?

• How would these practices give you more control over the discussion?

• Why is connecting important?

• What is the teacher’s role in helping students make connections?
NOAH’S ARK

• Can you figure out how many SEALs are needed to balance the 3rd deck?

• On each deck the animals on the left exactly balance the animals on the right.
PROBLEM SOLVING TEACHING TIPS

• “NEVER” tell an answer

• Teach by ASKING rather than teach by TELLING

• Make students justify EVERYTHING
  
  • “I don’t know, why do you think?”
  
  • “Convince me.”
  
  • “How do you know?”
  
  • “Please explain.”

• Be their PEER in problem solving
STUDENTS LEARN BEST BY MAKING SENSE OF THINGS THEMSELVES
You are buying supplies to make s’mores. You spend $12 on graham crackers. How much money did you start with?
MAKE A TABLE

• The first four rows of a pattern of stars are given as shown. How many stars will there be in 20th row?
GO TO EXTREMES

- When purchasing a bottle of water, which is a better deal: getting 1/3 more water at regular price OR getting the regular bottle at 1/3 off the price?
Can you connect a 3×3 square grid of dots using four straight lines without picking up your pen?
• What is the ones-digit of $2^{50}$?
SOLVE A SIMPLER PROBLEM

• How many squares are on a chess board?
WISHFUL THINKING

• When purchasing a bottle of water, which is a better deal: getting 1/3 more water at regular price OR getting the regular bottle at 1/3 off the price?
GUESS, CHECK AND REVISE

• A farm has cows and chickens. You see 18 heads and 50 feet. Can you figure out how many chickens and cows there are?