2-10 Perpendiculars and Distance

Copy each figure. Construct the segment that represents the distance indicated.
1. Y to TS

ANSWER:

SOLUTION:
The shortest distance from point Y to line TS is the length of a segment perpendicular to TS from point Y. Draw a segment from Y to TS.

ANSWER:

2. C to AB

SOLUTION:
The shortest distance from point C to line AB is the length of a segment perpendicular to AB from point C. Draw a perpendicular segment from C to AB.

ANSWER:

3. STRUCTURE After forming a line, every even member of a marching band turns to face the home team’s end zone and marches 5 paces straight forward. At the same time, every odd member turns in the opposite direction and marches 5 paces straight forward. Assuming that each band member covers the same distance, what formation should result? Justify your answer.

SOLUTION:
The formation should be that of two parallel lines that are also parallel to the 50-yard line; the band members have formed two lines that are equidistant from the 50-yard line, so by Theorem 3.9, the two lines formed are parallel.

ANSWER:
The formation should be that of two parallel lines that are also parallel to the 50-yard line; the band members have formed two lines that are equidistant from the 50-yard line, so by Theorem 3.9, the two lines formed are parallel.
COORDINATE GEOMETRY Find the distance from \( P \) to \( \ell \).

4. Line \( \ell \) contains points \((4, 3)\) and \((-2, 0)\). Point \( P \) has coordinates \((3, 10)\).

**SOLUTION:**
Use the slope formula to find the slope of the line \( \ell \).
Let \((x_1, y_1) = (4, 3)\) and \((x_2, y_2) = (-2, 0)\).

\[
\begin{align*}
m &= \frac{y_2 - y_1}{x_2 - x_1} \\
&= \frac{0 - 3}{-2 - 4} \\
&= \frac{-3}{-6} \\
&= \frac{1}{2}
\end{align*}
\]

Use the slope and any one of the points to write the equation of the line.

\[
y - y_1 = m(x - x_1) \quad \text{Point-Slope form}
\]

\[
y - 0 = \frac{1}{2}(x - (-2)) \quad \text{Substitution}
\]

\[
y = \frac{1}{2}x + 1 \quad \text{Equation 1}
\]

The slope of an equation perpendicular to \( \ell \) will be \(-2\). So, write the equation of a line perpendicular to \( \ell \) and that passes through \((3, 10)\).

\[
y - y_1 = m(x - x_1) \quad \text{Point-Slope form}
\]

\[
y - 10 = -2(x - 3) \quad \text{Substitution}
\]

\[
y - 10 + 10 = -2x + 6 + 10
\]

\[
y = -2x + 16 \quad \text{Equation 2}
\]

Solve the system of equations to determine the point of intersection.
The left sides of the equations are the same. So, equate the right sides and solve for \( x \).

\[
\frac{1}{2}x + 1 = -2x + 16 \quad \text{Equation 1} = \text{Equation 2}
\]

\[
\frac{1}{2}x + 2x = 16 - 1
\]

\[
\frac{5}{2}x = 15
\]

\[
x = 6
\]

Use the value of \( x \) to find the value of \( y \).

\[
y = -2x + 16 \quad \text{Equation 2}
\]

\[
y = -2(6) + 16 \quad \text{Substitution}
\]

\[
y = 4
\]

So, the point of intersection is \((6, 4)\).

Use the Distance Formula to find the distance between the points \((3, 10)\) and \((6, 4)\).

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

\[
d = \sqrt{(6 - 3)^2 + (4 - 10)^2}
\]

\[
d = \sqrt{9 + 36}
\]

\[
d = \sqrt{45}
\]

Therefore, the distance between the line and the point is \(3\sqrt{5}\) units.

**ANSWER:**

\(3\sqrt{5}\) units

5. Line \( \ell \) contains points \((-6, 1)\) and \((9, -4)\). Point \( P \) has coordinates \((4, 1)\).

**SOLUTION:**
Use the slope formula to find the slope of the line \( \ell \).
Let \((x_1, y_1) = (-6, 1)\) and \((x_2, y_2) = (9, -4)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]

\[
m = \frac{-4 - 1}{9 - (-6)}
\]

\[
m = \frac{-5}{15}
\]

\[
m = -\frac{1}{3}
\]

Use the slope and any one of the points to write the equation of the line. Let \((x_1, y_1) = (-6, 1)\).

\[
y - y_1 = m(x - x_1) \quad \text{Point-Slope form}
\]

\[
y - 1 = -\frac{1}{3}(x - (-6)) \quad \text{Substitution}
\]

\[
y = -\frac{1}{3}x + 2 + 1
\]

\[
y = -\frac{1}{3}x + 4 \quad \text{Equation 1}
\]

The slope of an equation perpendicular to \( \ell \) will be \(3\). So, write the equation of a line perpendicular to \( \ell \) and that passes through \((4, 1)\).
2-10 Perpendiculares and Distance

\[ y' - y_1 = m(x - x_1) \quad \text{Point-Slope form} \]
\[ y - 1 = 3(x - 4) \quad \text{Substitution} \]
\[ y = 3x - 12 + 1 \]
\[ y = 3x - 11 \quad \text{Equation 2} \]

Solve the system of equations to determine the point of intersection.
The left sides of the equations are the same. So, equate the right sides and solve for \( x \).
\[-\frac{1}{3}x - 1 = 3x - 11 \quad \text{Equation 1} = \text{Equation 2} \]
\[-\frac{1}{3}x - 3x = -11 + 1 \]
\[-\frac{10}{3}x = -10 \]
\[ x = 3 \quad \text{s-coord of pt of intersection} \]

Use the value of \( x \) to find the value of \( y \).
\[ y = 3x - 11 \quad \text{Equation 2} \]
\[ = 3(3) - 11 \quad \text{Substitution} \]
\[ = -2 \quad \text{y-coord of pt of intersection} \]
So, the point of intersection is \((3, -2)\).

Use the Distance Formula to find the distance between the points \((4, 1)\) and \((3, -2)\). Let \((x_1, y_1) = (4, 1)\) and \((x_2, y_2) = (3, -2)\).
\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]
\[ = \sqrt{(3 - 4)^2 + (-2 - 1)^2} \]
\[ = \sqrt{1 + 9} \]
\[ = \sqrt{10} \]
Therefore, the distance between the line and the point is \(\sqrt{10} \) units.

**ANSWER:**
\(\sqrt{10}\) units

6. Line \( \ell \) contains points \((4, 18)\) and \((-2, 9)\). Point \( P \) has coordinates \((-9, 5)\).

**SOLUTION:**
Use the slope formula to find the slope of the line \( \ell \).
Let \((x_1, y_1) = (4, 18)\) and \((x_2, y_2) = (-2, 9)\).

\[ \frac{y_2 - y_1}{x_2 - x_1} = m \quad \text{Slope} \]
\[ = \frac{9 - 18}{-2 - 4} \]
\[ = \frac{-9}{-6} \]
\[ = \frac{3}{2} \]

Use the slope and any one of the points to write the equation of the line. Let \((x_1, y_1) = (4, 18)\).
\[ y - y_1 = m(x - x_1) \quad \text{Point-Slope form} \]
\[ y - 18 = \frac{3}{2}(x - 4) \quad \text{Substitution} \]
\[ y - 18 = \frac{3}{2}x - 6 \]
\[ y - 18 - 18 = \frac{3}{2}x - 6 + 18 \]
\[ y = \frac{3}{2}x + 12 \quad \text{Equation 1} \]

The slope of an equation perpendicular to \( \ell \) will be \(-\frac{2}{3}\). So, write the equation of a line perpendicular to \( \ell \) and that passes through \((-9, 5)\).
\[ y - y_1 = m(x - x_1) \quad \text{Point-Slope form} \]
\[ y - 5 = -\frac{2}{3}(x - (-9)) \quad \text{Substitution} \]
\[ y - 5 = -\frac{2}{3}x - \frac{2}{3}(-9) \]
\[ y - 5 = -\frac{2}{3}x + 6 \]
\[ y - 5 = -\frac{2}{3}x - 6 + 5 \]
\[ y = -\frac{2}{3}x - 1 \quad \text{Equation 2} \]

Solve the system of equations to determine the point of intersection.
The left sides of the equations are the same. So, equate the right sides and solve for \( x \).
\[-\frac{2}{3}x - 1 = -\frac{2}{3}x + 12 \quad \text{Equation 2} = \text{Equation 3} \]
\[-\frac{2}{3}x - \frac{2}{3}x - 1 = -\frac{2}{3}x - \frac{2}{3}x + 12 \]
\[-\frac{2}{3}x - \frac{2}{3}x - 1 + 1 = -\frac{2}{3}x - \frac{2}{3}x + 12 + 1 \]
\[-\frac{13}{6}x = 13 \]
\[x = -6 \quad \text{s-coord of the pt of intersection} \]

Use the value of \( x \) to find the value of \( y \).
2-10 Perpendiculans and Distance

\[ y = \frac{3}{2}x + 12 \quad \text{Equation 1} \]
\[ y = \frac{3}{2}(-6) + 12 = 3 \quad \text{y-coord of pt of intersection} \]

So, the point of intersection is \((-6, 3)\).

Use the Distance Formula to find the distance between the points \((-9, 5)\) and \((-6, 3)\). Let \((x_1, y_1) = (-9, 5)\) and \((x_2, y_2) = (-6, 3)\).

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]
\[ = \sqrt{(-6 - (-9))^2 + (3 - 5)^2} \]
\[ = \sqrt{9 + 4} \]
\[ = \sqrt{13} \]

Therefore, the distance between the line and the point is \(\sqrt{13}\) units.

**ANSWER:**
\(\sqrt{13}\) units

**Find the distance between each pair of parallel lines with the given equations.**

7. \[ y = -2x + 4 \]
\[ y = -2x + 14 \]

**SOLUTION:**
To find the distance between the parallel lines, we need to find the length of the perpendicular segment between the two parallel lines. Pick a point on one of the equations, and write the equation of a line perpendicular through that point. Then use this perpendicular line and other equation to find the point of intersection. Find the distance between the two point using the distance formula.

Step 1: Find the equation of the line perpendicular to each of the lines.

\[ y = -2x + 4 \quad \text{Equation 1} \]
\[ y = -2x + 14 \quad \text{Equation 2} \]

The slope of a line perpendicular to both the lines will be \(\frac{1}{2}\). Consider the \(y\)-intercept of any of the two lines and write the equation of the perpendicular line through it. The \(y\)-intercept of the line \(y = -2x + 4\) is \((0, 4)\). So, the equation of a line with slope \(\frac{1}{2}\) and a \(y\)-intercept of 4 is \(y = \frac{1}{2}x + 4\). **Equation 3**

Step 2: Find the intersections of the perpendicular line and each of the other lines.

To find the point of intersection of the perpendicular and the second line, solve the two equations.

\[ -2x + 14 - \frac{1}{2}x + 4 \quad \text{Equation 2 = Equation 3} \]
\[ -2x - \frac{3}{2}x + 14 = \frac{1}{2}x - \frac{3}{2}x + 4 + 4 \]
\[ -\frac{15}{2}x + 14 = 8 \]
\[ -\frac{15}{2}x = -6 \]
\[ -\frac{15}{2}x + 6 = -\frac{15}{2}(-6) \]

\[ x = 6 \quad \text{x-coord of pt of intersection} \]

Use the value of \(x\) to find the value of \(y\).

\[ y = -2x + 14 \quad \text{Equation 2} \]
\[ = -2(6) + 14 \]
\[ = 6 \quad \text{y-coord of pt of intersection} \]

So, the point of intersection is \((4, 6)\).

Step 3: Find the length of the perpendicular between points.

Use the Distance Formula to find the distance between the points \((4, 6)\) and \((0, 4)\). Let \((x_1, y_1) = (4, 6)\) and \((x_2, y_2) = (0, 4)\).

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]
\[ = \sqrt{(4 - 6)^2 + (4 - 0)^2} \]
\[ = \sqrt{4 + 16} \]
\[ = \sqrt{20} \]
\[ = 2\sqrt{5} \]

Therefore, the distance between the two lines is \(2\sqrt{5}\) units.

**ANSWER:**
\(2\sqrt{5}\) units
8. \( y = 7 \)
\( y = -3 \)

**SOLUTION:**

The two lines have the coefficient of \( x \), zero. So, the slopes are zero. Therefore, the lines are horizontal lines passing through \( y = 7 \) and \( y = -3 \) respectively. The line perpendicular will be vertical. Thus, the distance, is the difference in the \( y \)-intercepts of the two lines. Then perpendicular distance between the two horizontal lines is \( 7 - (-3) = 10 \) units.

**ANSWER:**

10 units

---

9. \( Q \) to \( RS \)

**SOLUTION:**

The shortest distance from point \( Q \) to line \( RS \) is the length of a segment perpendicular to \( RS \) from point \( Q \). Draw a perpendicular segment from \( Q \) to \( RS \).

**ANSWER:**

---

10. \( A \) to \( BC \)

**SOLUTION:**

The shortest distance from point \( A \) to line \( BC \) is the length of a segment perpendicular to \( BC \) from point \( A \). Draw a perpendicular segment from \( A \) to \( BC \).

**ANSWER:**
11. $H$ to $FG$

**SOLUTION:**

The shortest distance from point $H$ to line $FG$ is the length of a segment perpendicular to $FG$ from point $H$. Draw a perpendicular segment from $H$ to $FG$.

**ANSWER:**

12. $K$ to $LM$

**SOLUTION:**

The shortest distance from point $K$ to line $LM$ is the length of a segment perpendicular to $LM$ from point $K$. Draw a perpendicular segment from $K$ to $LM$.

**ANSWER:**
13. **DRIVEWAYS** In the diagram, is the driveway shown the shortest possible one from the house to the road? Explain why or why not.

**SOLUTION:**
A driveway perpendicular to the road would be the shortest. The angle the driveway makes with the road is less than 90°, so it is not the shortest possible driveway.

**ANSWER:**
No; a driveway perpendicular to the road would be the shortest. The angle the driveway makes with the road is less than 90°, so it is not the shortest possible driveway.

14. **MODELING** Rondell is crossing the courtyard in front of his school. Three possible paths are shown in the diagram. Which of the three paths shown is the shortest? Explain your reasoning.

**SOLUTION:**
The shortest possible distance would be the perpendicular distance from one side of the courtyard to the other. Since Path B is the closest to 90°, it is the shortest of the three paths shown.

**ANSWER:**
Path B: The shortest possible distance would be the perpendicular distance from one side of the courtyard to the other. Since Path B is the closest to 90°, it is the shortest of the three paths shown.

**COORDINATE GEOMETRY** Find the distance from \( P \) to \( \ell \).

15. Line \( \ell \) contains points \((0, -3)\) and \((7, 4)\). Point \( P \) has coordinates \((4, 3)\).

**SOLUTION:**
Use the slope formula to find the slope of the line \( \ell \).
Let \((x_1, y_1) = (0, -3)\) and \((x_2, y_2) = (7, 4)\).
\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-3)}{7 - 0} = \frac{7}{7} = 1
\]
Use the slope and any one of the points to write the equation of the line. Let \((x_1, y_1) = (0, -3)\).
\[
y - y_1 = m(x - x_1) \quad \text{Point-Slope form}
\]
\[
y - (-3) = 1(x - 0) \quad \text{Substitution}
\]
\[
y + 3 = x
\]
\[
y + 3 - 3 = x - 3
\]
\[
y = x - 3 \quad \text{Equation 1}
\]
The slope of an equation perpendicular to \( \ell \) will be \(-1\). So, write the equation of a line perpendicular to \( \ell \) and that passes through \((4, 3)\).
\[
y - y_1 = m(x - x_1) \quad \text{Point-Slope form}
\]
\[
y - 3 = -1(x - 4) \quad \text{Substitution}
\]
\[
y - 3 = -x + 4
\]
\[
y - 3 + 3 = -x + 4 + 3
\]
\[
y = -x + 7 \quad \text{Equation 2}
\]
Solve the system of equations to determine the point of intersection.
\[
-x + 7 = x - 3 \quad \text{Equation 2 = Equation 1}
\]
\[-x - x + 7 = x - x - 3\]
\[-2x + 7 = -3 - 7\]
\[-2x = -10\]
\[-2x = -10\]
\[x = 5 \quad \text{x-coord. of pt. of intersection}
\]
Use the value of \( x \) to find the value of \( y \).
\[
y = x - 3 \quad \text{Equation 1}
\]
\[
y = 5 - 3 \quad \text{Substitution}
\]
\[y = 2 \quad \text{y-coord. pt. of intersection}
\]
So, the point of intersection is \((5, 2)\).

Use the Distance Formula to find the distance
2-10 Perpendiculars and Distance

between the points (5, 2) and (4, 3). Let \((x_1, y_1) = (5, 2)\) and \((x_2, y_2) = (4, 3)\).

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]
\[
= \sqrt{(4 - 5)^2 + (3 - 2)^2}
\]
\[
= \sqrt{1 + 1}
\]
\[
= \sqrt{2}
\]

Therefore, the distance between the line and the point is \(\sqrt{2}\) units.

**ANSWER:**

\(\sqrt{2}\) units

16. Line \(l\) contains points (11, -1) and (-3, -11). Point \(P\) has coordinates (-1, 1).

**SOLUTION:**

Use the slope formula to find the slope of the line \(l\). Let \((x_1, y_1) = (11, -1)\) and \((x_2, y_2) = (-3, -11)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]
\[
= \frac{-11 - (-1)}{-3 - 11}
\]
\[
= \frac{-10}{-14}
\]
\[
= \frac{5}{7}
\]

Use the slope and any one of the points to write the equation of the line. Let \((x_1, y_1) = (11, -1)\)

**Point-Slope form**

\[
y - y_1 = m(x - x_1)
\]
\[
y - (-1) = \frac{5}{7}(x - 11)
\]

**Substitution**

\[
y + 1 = \frac{5}{7}x - \frac{55}{7}
\]
\[
y + 1 - 1 = \frac{5}{7}x - \frac{55}{7} - 1
\]
\[
y = \frac{5}{7}x - \frac{62}{7}
\]

**Equation 1**

The slope of an equation perpendicular to \(l\) will be \(-\frac{7}{5}\). So, write the equation of a line perpendicular to \(l\) and that passes through (-1, 1).

**Point-Slope form**

\[
y - y_1 = m(x - x_1)
\]
\[
y - 1 = -\frac{7}{5}(x - (-1))
\]

**Substitution**

\[
y - 1 = -\frac{7}{5}(x + 1)
\]
\[
y - 1 = -\frac{7}{5}x - \frac{7}{5}
\]
\[
y - 1 + 1 = -\frac{7}{5}x - \frac{7}{5} + 1
\]
\[
y = -\frac{7}{5}x + \frac{2}{5}
\]

**Equation 2**

Solve the system of equations to determine the point of intersection.

\[
-\frac{7}{5}x - \frac{7}{5} = -\frac{5}{7}x - \frac{62}{7}
\]
\[
-\frac{7}{5}x - \frac{7}{5} - \frac{5}{7}x = -\frac{62}{7}
\]
\[
-\frac{21}{35}x - \frac{42}{35} = -\frac{62}{7}
\]
\[
-\frac{21}{35}x - \frac{42}{35} + \frac{105}{35} = -\frac{62}{7} + \frac{105}{35}
\]
\[
-\frac{21}{35}x + \frac{63}{35} = -\frac{26}{35}
\]
\[
x = \frac{-26}{-21}
\]
\[
x = \frac{26}{21}
\]

**x-coordinate of pt of intersection**

So, the point of intersection is (4, -6).

Use the Distance Formula to find the distance between the points (-1, 1) and (4, -6). Let \((x_1, y_1) = (-1, 1)\) and \((x_2, y_2) = (4, -6)\).

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]
\[
= \sqrt{(4 - (-1))^2 + (-6 - 1)^2}
\]
\[
= \sqrt{25 + 49}
\]
\[
= \sqrt{74}
\]

Therefore, the distance between the line and the point is \(\sqrt{74}\) units.

**ANSWER:**

\(\sqrt{74}\) units
17. Line \( \ell \) contains points \((-2, 1)\) and \((4, 1)\). Point \( P \) has coordinates \((5, 7)\).

**SOLUTION:**

Use the slope formula to find the slope of the line \( \ell \). Let \((x_1, y_1) = (-2, 1)\) and \((x_2, y_2) = (4, 1)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 1}{4 - (-2)} = \frac{0}{6} = 0
\]

Use the slope and any one of the points to write the equation of the line. Let \((x_1, y_1) = (-2, 1)\).

\[
y - y_1 = m(x - x_1)
\]

**Point-Slope form**

\[
y - 1 = 0(x - (-2))
\]

**Substitution**

\[
y - 1 = 0
\]

\[
y - 1 + 1 = +1
\]

\[
y = 1
\]

The slope of an equation perpendicular to \( \ell \) will be undefined, and hence the line will be a vertical line.

The equation of a vertical line through \((5, 7)\) is \(x = 5\). The point of intersection of the two lines is \((5, 1)\).

Use the Distance Formula to find the distance between the points \((5, 1)\) and \((5, 7)\). Let \((x_1, y_1) = (5, 1)\) and \((x_2, y_2) = (5, 7)\).

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

\[
= \sqrt{(5 - 5)^2 + (7 - 1)^2}
\]

\[
= \sqrt{0 + 36}
\]

\[
= 6
\]

Therefore, the distance between the line and the point is 6 units.

**ANSWER:**

6 units

18. Line \( \ell \) contains points \((4, -1)\) and \((4, 9)\). Point \( P \) has coordinates \((1, 6)\).

**SOLUTION:**

Use the slope formula to find the slope of the line \( \ell \). Let \((x_1, y_1) = (4, -1)\) and \((x_2, y_2) = (4, 9)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - (-1)}{4 - 4} = \frac{10}{0}
\]

Division by zero is undefined. So, the slope of the line is undefined and the line is a vertical line through \((4, -1)\) and \((4, 9)\). So, the equation of the line is \(x = 4\).

A line perpendicular to \( \ell \) will be a horizontal line. Horizontal lines have zero slopes. The equation of a horizontal line through \((1, 6)\) is \(y = 6\).

The point of intersection of the two lines is \((4, 6)\).

Use the Distance Formula to find the distance between the points \((4, 6)\) and \((1, 6)\).

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

\[
= \sqrt{(1 - 4)^2 + (6 - 6)^2}
\]

\[
= \sqrt{9 + 0}
\]

\[
= 3
\]

Therefore, the distance between the line and the point is 3 units.

**ANSWER:**

3 units

19. Line \( \ell \) contains points \((1, 5)\) and \((4, -4)\). Point \( P \) has coordinates \((-1, 1)\).

**SOLUTION:**

Use the slope formula to find the slope of the line \( \ell \). Let \((x_1, y_1) = (1, 5)\) and \((x_2, y_2) = (4, -4)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 5}{4 - 1} = \frac{-9}{3} = -3
\]

Use the slope and any one of the points to write the
2-10 Perpendiculars and Distance

The slope of an equation perpendicular to \( \ell \) will be \( \frac{1}{3} \). So, write the equation of a line perpendicular to \( \ell \) and that passes through \((-1, 1)\).

\[
y - y_1 = m(x - x_1) \quad \text{Point-Slope form}
\]

\[
y - 1 = \frac{1}{3}(x - (-1)) \quad \text{Substitution}
\]

\[
y - 1 = \frac{1}{3}x + \frac{1}{3}
\]

\[
y - 1 + 1 = \frac{1}{3}x + \frac{1}{3} + 1
\]

\[
y = \frac{1}{3}x + \frac{1}{3} + \frac{2}{3}
\]

\[
y = \frac{1}{3}x + \frac{4}{3} \quad \text{Equation 2}
\]

Solve the system of equations to determine the point of intersection.

The left sides of the equations are the same. So, equate the right sides and solve for \(x\).

\[
\frac{1}{3}x + \frac{4}{3} = -3x + 8 \quad \text{Equation 2 = Equation 1.}
\]

\[
\frac{1}{3}x + 3x + \frac{4}{3} = -3x + 8
\]

\[
\frac{1}{3}x + \frac{2}{3} = 8
\]

\[
\frac{10}{3}x + 4 - \frac{2}{3} = \frac{24}{3} - \frac{4}{3}
\]

\[
\frac{10}{3}x = \frac{20}{3}
\]

\[
\frac{3}{10} (\frac{10}{3}x) = \frac{3}{10} (\frac{20}{3})
\]

\[
x = 2 \quad \text{x-coord of pt of intersection}
\]

Use the value of \(x\) to find the value of \(y\).

\[
y = -3x + 8 \quad \text{Equation 1}
\]

\[
y = -3(2) + 8
\]

\[
y = 2 \quad \text{y-coord of pt of intersection}
\]

So, the point of intersection is \((2, 2)\).

Use the Distance Formula to find the distance between the points \((2, 2)\) and \((-1, 1)\).
20. Line \( \ell \) contains points \((-8, 1)\) and \((3, 1)\). Point \( P \) has coordinates \((-2, 4)\).

**SOLUTION:**
Use the slope formula to find the slope of the line \( \ell \). Let \((x_1, y_1) = (-8, 1)\) and \((x_2, y_2) = (3, 1)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 1}{3 - (-8)} = \frac{0}{11} = 0
\]

Use the slope and any one of the points to write the equation of the line. Let \((x_1, y_1) = (-8, 1)\).

\[
y - y_1 = m(x - x_1) \quad \text{Point-Slope form}
\]
\[
y - 1 = 0(x - (-8))
\]
\[
y - 1 = 0 \quad \text{Equation 1}
\]

The slope of an equation perpendicular to \( \ell \) will be undefined, or the line will be a vertical line. The equation of a vertical line through \((-2, 4)\) is \( x = -2 \). The point of intersection of the two lines is \((-2, 1)\).

Use the Distance Formula to find the distance between the points \((-2, 1)\) and \((-2, 4)\). Let \((x_1, y_1) = (-2, 1)\) and \((x_2, y_2) = (-2, 4)\).

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]
\[
= \sqrt{(-2 - (-2))^2 + (4 - 1)^2}
\]
\[
= \sqrt{0 + 9}
\]
\[
= 3
\]

Therefore, the distance between the line and the point is 3 units.

**ANSWER:**
3 units

Find the distance between each pair of parallel lines with the given equations.

21. \( y = -2 \)

\( y = 4 \)

**SOLUTION:**

![Diagram showing two horizontal lines with a distance of 6 units between them.]

The two lines are horizontal lines and for each equation, the coefficient of \( x \)-term is zero. So, the slopes are zero. Therefore, the line perpendicular to the parallel lines will be vertical.

The distance of the vertical line between the parallel lines, will be the difference in the \( y \)-intercepts. To find the perpendicular distance between the two horizontal lines subtract \(-2\) from \(4\) to get \(4 - (-2) = 6\) units.

**ANSWER:**
6 units
2-10 Perpendiculars and Distance

22. $x = 3$

SOLUTION:

![Graph showing two vertical lines with the points $x = 3$ and $x = 7$.]

The two lines are vertical and of the form $x = a$. So, the slopes are undefined. Therefore, the lines are vertical lines passing through $x = 3$ and $x = 7$ respectively. The line perpendicular to each line will be horizontal. The distance will be the difference in the $x$-intercepts. To find the perpendicular distance between the two horizontal lines subtract 3 from 7 to get $7 - 3 = 4$ units.

ANSWER:

4 units

23. $y = 5x - 22$

$y = 5x + 4$

SOLUTION:

To find the distance between the parallel lines, we need to find the length of the perpendicular segment between the two parallel lines. Pick a point on one of the equation, and write the equation of a line perpendicular through that point. Then use this perpendicular line and other equation to find the point of intersection. Find the distance between the two points using the distance formula.

Step 1: Find the equation of the line perpendicular to each of the lines.

$y = 5x - 22$  Equation 1

$y = 5x + 4$  Equation 2

The slope of a line perpendicular to both the lines will be $-\frac{1}{5}$. Consider the $y$-intercept of any of the two lines and write the equation of the perpendicular line through it. The $y$-intercept of the line $y = 5x + 4$ is $(0, 4)$. So, the equation of a line with slope $-\frac{1}{5}$ and a $y$-intercept of 4 is $y = -\frac{1}{5}x + 4$.  Equation 3

Step 2: Find the intersections of the perpendicular line and each of the other lines.

To find the point of intersection of the perpendicular and the second line, solve the two equations. The left sides of the equations are the same. So, equate the right sides and solve for $x$.

$5x - 22 = -\frac{1}{5}x + 4$  Equation 1 = Equation 3

$5x + \frac{1}{5}x - 22 = -\frac{1}{5}x + \frac{1}{5}x + 4$

$\frac{25}{5}x + \frac{1}{5}x - 22 = 4$

$\frac{26}{5}x - 22 + 4 = 22 + 4$

$\frac{26}{5}x - 26$

$\frac{26}{5} \cdot \frac{5}{x} = \frac{26}{26}$

$x = 5$

Use the value of $x$ to find the value of $y$.

$y = 5(5) - 22$  Equation 1

$= 25 - 22$

$= 3$  y-coord of pt. of intersection

So, the point of intersection is $(5, 3)$.

Step 3: Find the length of the perpendicular between points

Use the Distance Formula to find the distance between the points $(5, 3)$ and $(0, 4)$. Let $(x_1, y_1) = (5, 3)$ and $(x_2, y_2) = (0, 4)$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(0 - 5)^2 + (4 - 3)^2}$

$= \sqrt{25 + 1}$

$= \sqrt{26}$

Therefore, the distance between the two lines is $\sqrt{26}$ units.

ANSWER:

$\sqrt{26}$ units
2-10 Perpendiculars and Distance

\[ y = \frac{1}{3} x - 3 \]

24. \[ y = \frac{1}{3} x + 2 \]

**SOLUTION:**
To find the distance between the parallel lines, we need to find the length of the perpendicular segment between the two parallel lines. Pick a point on one of the equations and write the equation of a line perpendicular through that point. Then use this perpendicular line and other equation to find the point of intersection. Find the distance between the two points using the distance formula.

**Step 1:** Find the equation of the line perpendicular to each of the lines.

\[ y = \frac{1}{3} x - 3 \quad \text{Equation 1} \]

\[ y = \frac{1}{3} x + 2 \quad \text{Equation 2} \]

The slope of a line perpendicular to both the lines will be \(-3\). Consider the y-intercept of any of the two lines and write the equation of the perpendicular line through it. The y-intercept of the line \( y' = \frac{1}{3} x + 2 \) is \((0, 2)\). So, the equation of a line with slope \(-3\) and a y-intercept of 2 is

\[ y = -3x + 2. \quad \text{Equation 3} \]

**Step 2:** Find the intersections of the perpendicular line and each of the other lines.

To find the point of intersection of the perpendicular and the second line, solve the two equations.

The left sides of the equations are the same. So, equate the right sides and solve for \( x \).

\[-3x + 2 = \frac{1}{3} x - 3 \quad \text{Equation 2 = Equation 3} \]

\[ -3x - \frac{1}{3} x + 2 = \frac{1}{3} x - \frac{1}{3} x - 3 \]

\[ -\frac{9}{3} x + 2 = -3 \]

\[ -\frac{10}{3} x + 2 - 2 = -3 - 2 \]

\[ -\frac{10}{3} x = -5 \]

\[ -\frac{3}{10} \left( -\frac{10}{3} x \right) = -\frac{3}{10} \left( -5 \right) \]

\[ x = \frac{3}{2} \]

\[ x = \frac{3}{2} \quad \text{x-coord of pt. of intersection} \]

Use the value of \( x \) to find the value of \( y \).

\[ y = \frac{1}{3} \left( \frac{3}{2} \right) - 3 \]

\[ = \frac{1}{3} \left( \frac{3}{2} \right) - 3 \]

\[ = -\frac{5}{2} \]

\[ = -2 \frac{1}{2} \quad \text{y-coord of pt. of intersection} \]

So, the point of intersection is \( \left( \frac{1}{2}, -2 \frac{1}{2} \right) \).

**Step 3:** Find the length of the perpendicular between points.

Use the Distance Formula to find the distance between the points \( \left( \frac{1}{2}, -2 \frac{1}{2} \right) \) and \((0, 2)\). Let \( x_1, y_1 \) = \( \left( \frac{1}{2}, -2 \frac{1}{2} \right) \) and \((x_2, y_2) = (0, 2)\).

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

\[ = \sqrt{(0 - \frac{3}{2})^2 + (2 - (\frac{5}{2}))^2} \]

\[ = \sqrt{\frac{9}{4} + \frac{81}{4}} \]

\[ = \sqrt{\frac{90}{4}} \]

\[ = \frac{3}{2} \sqrt{10} \]

Therefore, the distance between the two lines is \( 1.5 \sqrt{10} \) units.

**ANSWER:**

\( 1.5 \sqrt{10} \) units
**2-10 Perpendiculars and Distance**

25. \( x = 8.5 \)
\( x = -12.5 \)

**SOLUTION:**

The two lines are vertical and of the form \( x = a \). So, the slopes are undefined. Therefore, the lines are vertical lines passing through \( x = 8.5 \) and \( x = -12.5 \) respectively.

The line perpendicular to each line will be horizontal. The distance will be the difference in the \( x \)-intercepts. To find the perpendicular distance between the two horizontal lines subtract \(-12.5\) from \(8.5\) to get \(8.5 - (-12.5) = 21\) units.

**ANSWER:**

21 units

26. \( y = 15 \)
\( y = -4 \)

**SOLUTION:**

The two lines are horizontal and each equation has a coefficient of zero for the \( x \)-term. So, the slopes are zero. Therefore, the lines are horizontal lines passing through \( y = 15 \) and \( y = -4 \) respectively. The line perpendicular to each line will be vertical. The distance will be the difference in the \( y \)-intercepts. To find the perpendicular distance between the two horizontal lines subtract \(-4\) from 15 to get \(15 - (-4) = 19\) units.

**ANSWER:**

19 units

27. \( y = \frac{1}{4}x + 2 \)
\( 4y - x = -60 \)

**SOLUTION:**

To find the distance between the parallel lines, we need to find the length of the perpendicular segment between the two parallel lines. Pick a point on one of the equation, and write the equation of a line perpendicular through that point. Then use this perpendicular line and other equation to find the point of intersection. Find the distance between the two point using the distance formula.

Step 1: Find the equation of the line perpendicular to each of the lines.
First, write the second equation also in the slope-intercept form.
\( 4y - x = -60 \)
\( 4y = x - 60 \)
\( y = \frac{1}{4}x - 15 \) \textbf{Equation 2}
2-10 Perpendiculars and Distance

\[ y = \frac{1}{4}x + 2 \quad \text{Equation 1} \]

The slope of a line perpendicular to both the lines will be \(-4\). Consider the \(y\)-intercept of any of the two lines and write the equation of the perpendicular line through it. The \(y\)-intercept of the line \( y = \frac{1}{4}x + 2 \) is (0, 2). So, the equation of a line with slope \(-4\) and a \(y\)-intercept of 2 is \( y = -4x + 2 \). \textbf{Equation 3}

Step 2: Find the intersections of the perpendicular line and each of the other lines.

To find the point of intersection of the perpendicular and the second line, solve the two equations. The left sides of the equations are the same. So, equate the right sides and solve for \(x\).

\[
-4x + 2 = \frac{1}{4}x - 15 \quad \text{Equation 2}
\]

\[
-4x - 2\frac{1}{4}x + 2 = -\frac{1}{4}x - 15
\]

\[
-16x - \frac{3}{4}x + 2 - 15
\]

\[
-12x + 2 - 15 - 2
\]

\[
-12x = -17
\]

\[
\frac{4}{17}\left(\frac{17}{4}x\right) = \frac{4}{17}(-17)
\]

\[
\frac{4}{17} = x = -4
\]

\text{Point at pt of intersection}

Use the value of \(x\) to find the value of \(y\).

\[
y = \frac{1}{4}x - 15 \quad \text{Equation 2}
\]

\[
y = \frac{1}{4}(4) = 1 - 15
\]

\[
y = -14 \quad \text{\(y\)-coordinate of point of intersection}
\]

So, the point of intersection is (4, -14).

Step 3: Find the length of the perpendicular between points.

Use the Distance Formula to find the distance between the points (4, -14) and (0, 2). Let \((x_1,y_1) = (4, -14)\) and \((x_2,y_2) = (0, 2)\).

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

\[
d = \sqrt{(0 - 4)^2 + (2 - (-14))^2}
\]

\[
d = \sqrt{16 + 256}
\]

\[
d = 4\sqrt{17}
\]

Therefore, the distance between the two lines is \(4\sqrt{17}\) units.

**ANSWER:**

\(4\sqrt{17}\) units

28. \(3x + y = 3\)

\(y + 17 = -3x\)

**SOLUTION:**

To find the distance between the parallel lines, we need to find the length of the perpendicular segment between the two parallel lines. Pick a point on one of the equation, and write the equation of a line perpendicular through that point. Then use this perpendicular line and other equation to find the point of intersection. Find the distance between the two point using the distance formula.

Step 1: Find the equation of the line perpendicular to each of the lines.

First, write the two equations in slope-intercept form.

\[
3x + y = 3
\]

\[
y = -3x + 3 \quad \text{Equation 1}
\]

\[
y = -3x - 17 \quad \text{Equation 2}
\]

The slope of a line perpendicular to both the lines will be \(\frac{1}{3}\). Consider the \(y\)-intercept of any of the two lines and write the equation of the perpendicular line through it. The \(y\)-intercept of the line \(y = -3x + 3\) is (0, 3). So, the equation of a line with slope \(\frac{1}{3}\) and a \(y\)-intercept of 3 is

\[
y = \frac{1}{3}x + 3 \quad \text{Equation 3}
\]

Step 2: Find the intersections of the perpendicular line and each of the other lines.

To find the point of intersection of the perpendicular and the second line, solve the two equations. The left sides of the equations are the same. So, equate the right sides and solve for \(x\).

\[
-3x - 17 = \frac{1}{3}x + 3
\]

\[
-3x - 1\frac{1}{3}x - 17 = \frac{1}{3}x + 3
\]

\[
-\frac{10}{3}x - 17 + 17 = 3 + 17
\]

\[
-\frac{10}{3}x = 20
\]

\[
\frac{10}{3}x = -6 \quad \text{point at pt of intersection}
\]

Use the value of \(x\) to find the value of \(y\).

\[
y = -3x - 17 \quad \text{Equation 2}
\]

\[
y = -3(-6) - 17
\]

\[
y = 1 \quad \text{point at pt of intersection}
\]

So, the point of intersection is \((-6, 1)\).

Step 3: Find the length of the perpendicular between
2-10 Perpendiculars and Distance

points.
Use the Distance Formula to find the distance between the points \((-6, 1)\) and \((0, 3)\). 
\((x_1, y_1) = (-6, 1)\) and 
\((x_2, y_2) = (0, 3)\).
\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]
\[
= \sqrt{(0 - (-6))^2 + (3 - 1)^2}
\]
\[
= \sqrt{36 + 4}
\]
\[
= \sqrt{40}
\]
\[
= 2\sqrt{10}
\]
Therefore, the distance between the two lines is \(2\sqrt{10}\) units.

**ANSWER:** 
\(2\sqrt{10}\) units

29. \(y = -\frac{5}{4}x + 3.5\)

**SOLUTION:**
To find the distance between the parallel lines, we need to find the length of the perpendicular segment between the two parallel lines. Pick a point on one of the equation, and write the equation of a line perpendicular through that point. Then use this perpendicular line and other equation to find the point of intersection. Find the distance between the two point using the distance formula.

Step 1: Find the equation of the line perpendicular to each of the lines.
First, write the second equation also in the slope-intercept form.
\[
4y + 10.6 = -5x
\]
\[
4y = -5x - 10.6
\]
\[
y = -\frac{5}{4}x - 2.65 \quad \text{Equation 2}
\]
\[
y = -\frac{5}{4}x + 3.5 \quad \text{Equation 1}
\]
The slope of a line perpendicular to both the lines will be \(\frac{4}{5}\). Consider the \(y\)-intercept of any of the two lines and write the equation of the perpendicular line through it. The \(y\)-intercept of the line \(y = -\frac{5}{4}x + 3.5\) is \((0, 3.5)\). So, the equation of a line with slope \(\frac{4}{5}\) and a \(y\)-intercept of 3.5 is
\[
y = \frac{4}{5}x + 3.5 \quad \text{Equation 3}
\]

Step 2: Find the intersections of the perpendicular line and each of the other lines.
To find the point of intersection of the perpendicular and the second line, solve the two equations.
The left sides of the equations are the same. So, equate the right sides and solve for \(x\).
\[
-\frac{5}{4}x - 2.65 = \frac{4}{5}x + 3.5 \quad \text{Equation 3} = \text{Equation 1}
\]
\[
-\frac{5}{4}x - \frac{4}{5}x = 2.65 - 3.5
\]
\[
-\frac{25}{20}x - \frac{16}{20}x = -0.85
\]
\[
\frac{-41}{20}x = -0.85
\]
\[
\frac{-41}{20}\left(-\frac{41}{20}\right) = \frac{-20}{41}\left(0.12\right)
\]
\[
x = \frac{-3}{4}
\]
Use the value of \(x\) to find the value of \(y\).
\[
y = \frac{4}{5}x + 3.5 \quad \text{Equation 1}
\]
\[
y = \frac{4}{5}(-3) + 3.5
\]
\[
y = 1.1
\]
So, the point of intersection is \((-3, 1.1)\).

Step 3: Find the length of the perpendicular between points.
Use the Distance Formula to find the distance between the points \((-3, 1.1)\) and \((0, 3.5)\). Let \((x_1, y_1) = (-3, 1.1)\) and \((x_2, y_2) = (0, 3.5)\).
\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]
\[
= \sqrt{(0 - (-3))^2 + (3.5 - 1.1)^2}
\]
\[
= \sqrt{9 + 5.76}
\]
\[
= \sqrt{14.76}
\]
Therefore, the distance between the two lines is \(\sqrt{14.76}\) units.

**ANSWER:** 
\(\sqrt{14.76}\) units
30. **PROOF** Write a two-column proof of Theorem 2.24.

*SOLUTION:*

**Given:** \( \ell \) is equidistant to \( m \), and \( n \) is equidistant to \( m \).

**Prove:** \( \ell \parallel n \)

**Proof:**

**Statements (Reasons)**
1. \( \ell \) is equidistant to \( m \), and \( n \) is equidistant to \( m \). (Given)
2. \( \ell \parallel m \) and \( m \parallel n \) (Def. of equidistant)
3. slope of \( \ell \) = slope of \( m \), slope of \( m \) = slope of \( n \) (Def. of \( \parallel \) lines)
4. slope of \( \ell \) = slope of \( n \) (Substitution)
5. \( \ell \parallel n \) (Def. of \( \parallel \) lines)

**ANSWER:**

**Given:** \( \ell \) is equidistant to \( m \), and \( n \) is equidistant to \( m \).

**Prove:** \( \ell \parallel n \)

**Proof:**

**Statements (Reasons)**
1. \( \ell \) is equidistant to \( m \), and \( n \) is equidistant to \( m \). (Given)
2. \( \ell \parallel m \) and \( m \parallel n \) (Def. of equidistant)
3. slope of \( \ell \) = slope of \( m \), slope of \( m \) = slope of \( n \) (Def. of \( \parallel \) lines)
4. slope of \( \ell \) = slope of \( n \) (Substitution)
5. \( \ell \parallel n \) (Def. of \( \parallel \) lines)

**Find the distance from the line to the given point.**

31. \( y = -3 \), (5, 2)

*SOLUTION:*

![Graph showing the line \( y = -3 \) and a point (5, 2)](image)

The slope of an equation perpendicular to \( y = -3 \) will be undefined, or the line will be a vertical line. The equation of a vertical line through (5, 2) is \( x = 5 \). The point of intersection of the two lines is (5, -3).

Use the Distance Formula to find the distance between the points (5, 2) and (5, -3). Let \((x_1, y_1) = (5, 2)\) and \((x_2, y_2) = (5, -3)\).

\[
\begin{align*}
  d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
  &= \sqrt{(5 - 5)^2 + (-3 - 2)^2} \\
  &= \sqrt{0 + 25} \\
  &= 5
\end{align*}
\]

Therefore, the distance between the line and the point is 5 units.

**ANSWER:**

5 units
2-10 Perpendiculars and Distance

32. \( y = \frac{1}{6}x + 6, (-6,5) \)

**SOLUTION:**

The slope of an equation perpendicular to 
\( y = \frac{1}{6}x + 6 \) will be \(-6\). A line with a slope \(-6\) and 
that passes through the point \((-6, 5)\) will have the equation,

\[
\begin{align*}
y - 5 &= \frac{1}{6}(x + 6) \\
y - 5 &= \frac{1}{6}x + 1 \\
y &= \frac{1}{6}x + 6
\end{align*}
\]

Solve the two equations to find the point of intersection.

The left sides of the equations are the same. So, 
equate the right sides and solve for \(x\).

\[
\begin{align*}
\frac{1}{6}x + 6 &= -6x - 31 \\
\frac{1}{6}x + 6 &= -6x + 6x - 31 \\
\frac{1}{6}x + 6 &= -31 - 6 \\
\frac{37}{6}x &= -37 \\
x &= -6
\end{align*}
\]

Use the value of \(x\) to find the value of \(y\).

\[
\begin{align*}
y &= -6(-6) - 31 \\
&= 5
\end{align*}
\]

The point of intersection of the two lines is \((-6, 5)\).

Therefore, the distance between the line and the 
point is 0 units.

**ANSWER:** 0 units

33. \( x = 4, (-2, 5) \)

**SOLUTION:**

The slope of an equation perpendicular to \( x = 4 \) will 
be zero, or the line will be a horizontal line. The 
equation of a horizontal line through \((-2, 5)\) is \( y = 5 \).

The point of intersection of the two lines is \((4, 5)\). 
Use the Distance Formula to find the distance 
between the points \((4, 5)\) and \((-2, 5)\). Let \((x_1, y_1) = \( (4, 5) \) and \((x_2, y_2) = (-2, 5)\).

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

\[
= \sqrt{(-2 - 4)^2 + (5 - 5)^2}
\]

\[
= \sqrt{36 + 0}
\]

Therefore, the distance between the line and the 
point is 6 units.

**ANSWER:** 6 units
34. **POSTERS** Alma is hanging two posters on the wall in her room as shown. How can Alma use perpendicular distances to confirm that the posters are parallel?

![Posters Image]

**SOLUTION:**
Alma can measure the perpendicular distance between the posters in two different places as shown. If these distances are equal, then the posters are parallel.

**ANSWER:**
Alma can measure the perpendicular distance between the posters in two different places. If these distances are equal, then the posters are parallel.

35. **SCHOOL SPIRIT** Brock is decorating a hallway bulletin board to display pictures of students demonstrating school spirit. He cuts off one length of border to match the width of the top of the board, and then uses that strip as a template to cut a second strip that is exactly the same length for the bottom. When stapling the bottom border in place, he notices that the strip he cut is about a quarter of an inch too short. Describe what he can conclude about the bulletin board. Explain your reasoning.

![Bulletin Board Image]

**SOLUTION:**
He can conclude that the right and left sides of the bulletin board are not parallel, since the perpendicular distance between one line and any point on the other line must be equal be the same anywhere on the lines for the two lines to be parallel.

**ANSWER:**
He can conclude that the right and left sides of the bulletin board are not parallel, since the perpendicular distance between one line and any point on the other line must be equal be the same anywhere on the lines for the two lines to be parallel.

**CONSTRUCTION** Line \( \ell \) contains points at \((-4, 3)\) and \((2, -3)\). Point \(P\) at \((-2, 1)\) is on line \( \ell \). Complete the following construction.

**Step 1**
Graph line \( \ell \) and point \(P\), and put the compass at point \(P\). Using the same compass setting, draw arcs to the left and right of \(P\). Label these points \(A\) and \(B\).
2-10 Perpendicularrs and Distance

Step 2
Open the compass to a setting greater than $AP$. Put the compass at point $A$ and draw an arc above line $\ell$. Label the point of intersection $Q$. Then draw $PQ$.

36. What is the relationship between line $\ell$ and $PQ$? Verify your conjecture using the slopes of the two lines.

SOLUTION:
Sample answer: The lines are perpendicular; the slope of $\ell$ is $-1$ and the slope of $PQ$ is $1$. Since the slopes are negative reciprocals, the lines are perpendicular.

ANSWER:
Sample answer: The lines are perpendicular; the slope of $\ell$ is $-1$ and the slope of $PQ$ is $1$. Since the slopes are negative reciprocals, the lines are perpendicular.

37. Repeat the construction above using a different line and point on that line.

SOLUTION:
Sample answer:
Step 1:
Graph line through points $(-2,-4)$, $(2,-2)$, and $(4,-1)$ with Point P at $(-2,-4)$. Put the compass at point $P$. Using the same compass setting, draw arcs to the left and right of $P$. Label these points $A$ and $B$.

Step 2:
Open the compass to a setting greater than $AP$. Put the compass at point $A$ and draw an arc above the line.

Step 3:
Using the same compass setting, put the compass at point $B$ and draw an arc above the line. Label the point of intersection $Q$. Then draw a line through $Q$ and $P$.

ANSWER:
See students’ work.
2-10 Perpendiculars and Distance

38. **SENSE-MAKING** \( \overline{AB} \) has a slope of 2 and midpoint \( M(3, 2) \). A segment perpendicular to \( \overline{AB} \) has midpoint \( P(4, -1) \) and shares endpoint \( B \) with \( \overline{AB} \).
   a. Graph the segments.
   b. Find the coordinates of \( A \) and \( B \).

   **SOLUTION:**
   a. From the point \( M(3, 2) \), move 2 units up and 1 unit to the right to plot the point \( A(4, 4) \).
   There are two methods to find the coordinates of \( B \).

   **Method 1**
   Use the Midpoint Formula with the coordinates of \( A \) and \( M \).
   
   \[
   M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)
   \]
   
   \[
   (3, 2) = \left( \frac{x_1 + 2}{2}, \frac{y_1 + 0}{2} \right)
   \]
   
   \[
   \frac{x_1 + 2}{2} = 3 \Rightarrow x_1 = 2(3) - 2 = 4
   \]
   
   \[
   \frac{y_1 + 0}{2} = 2 \Rightarrow 2(2) - 0 = 4
   \]

   **Method 2**
   Use the slope.
   To get from \( A \) to \( M \) on the graph, move down 2 units and to the left 1 unit. Since \( M \) is the midpoint, \( B \) and \( A \) are equidistant from \( M \). From \( M \), move down 2 units and to the left 1 unit. \( B \) has the coordinates (2, 0).

   Next plot \( P \) on the graph. Since \( BP \) is perpendicular to \( AB \), the slope of \( BP \) is the negative reciprocal of the slope of \( AB \). Since the slope of \( AB \) is 2, the slope of \( BP \) is \(-\frac{1}{2}\). Use this slope to find and plot the other endpoint.

   **ANSWER:**
   a. 
   b. \( A(4, 4), B(2, 0) \)

b. From the graph in part a, the coordinates of \( A \) is (4, 4) and that of \( B \) is (2, 0).
39. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the areas of triangles formed by points on parallel lines.

**a. GEOMETRIC** Draw two parallel lines and label them as shown.

![Image of parallel lines](image)

**b. VERBAL** Where would you place point C on line m to ensure that triangle ABC would have the largest area? Explain your reasoning.

**c. ANALYTICAL** If AB = 11 inches, what is the maximum area of △ABC?

**SOLUTION:**

**a.** See students’ work.

**b.** Place point C any place on line m. The area of the triangle is \( \frac{1}{2} \) the height of the triangle times the length of the base of the triangle. Since the two lines are parallel, the distance between them will always be the same. The numbers stay constant regardless of the location of C on line m.

**c.** Substitute \( h = 3 \) and \( b = 11 \) in the formula to find the area.

\[
A = \frac{1}{2} (3)(11) = 16.5
\]

The maximum area of the triangle will be 16.5 in\(^2\).

**ANSWER:**

**a.** See students’ work.

**b.** Place point C any place on line m. The area of the triangle is \( \frac{1}{2} \) the height of the triangle times the length of the base of the triangle. The numbers stay constant regardless of the location of C on line m.

**c.** 16.5 in\(^2\)

40. **MULTI-STEP** Draw a diagram that represents each statement, marking the diagram with the given information. Then, use your diagram to answer the given questions.

- Lines a and b are perpendicular to plane P.
- Plane P is perpendicular to planes R and Q.
- Plane Q is perpendicular to line ℓ.

**a.** If planes R and Q are parallel and they intersect plane P, what must also be true?

**b.** If line ℓ is perpendicular to plane Q, what must also be true?

**c.** If both line a and line b are perpendicular to plane P, what must also be true?

**SOLUTION:**

**a.** Since the planes R and Q are parallel and they intersect plane P, the lines formed by the intersections are parallel.

**b.** Planes R and Q are parallel, so a line perpendicular to plane Q will also be perpendicular to plane R.

**c.** Line a and line b are perpendicular to plane P, so lines a and b are coplanar.

**ANSWER:**

**a.** The lines formed by the intersection of R and Q with P are parallel.

**b.** Line ℓ is perpendicular to plane R.

**c.** Lines a and b are coplanar.
41. **ERROR ANALYSIS** Han draws the segments \(\overline{AB}\) and \(\overline{CD}\) shown below using a straightedge. He claims that these two lines, if extended, will never intersect. Shenequa claims that they will. Is either of them correct? Justify your answer.

\[ A \quad B \quad C \quad D \]

**SOLUTION:**
When using the student edition, the answer will be: The distance between points \(A\) and \(C\) is 1.2 cm. The distance between points \(B\) and \(D\) is 1.35 cm. Since the lines are not equidistant everywhere, the lines will eventually intersect when extended. Therefore, Shenequa is correct.
For other forms of media, the answer will vary.

**ANSWER:**
When using the student edition, the answer will be: The distance between points \(A\) and \(C\) is 1.2 cm. The distance between points \(B\) and \(D\) is 1.35 cm. Since the lines are not equidistant everywhere, the lines will eventually intersect when extended. Shenequa is correct.
For other forms of media, the answer will vary.

42. **CHALLENGE** Describe the locus of points that are equidistant from two intersecting lines, and sketch an example.

**SOLUTION:**
\(\overline{AB}\) and \(\overline{CD}\) intersect at \(X\) to form 2 pairs of vertical angles. The locus of points equidistant from \(\overline{AB}\) and \(\overline{CD}\) lie along \(\overline{EF}\) and \(\overline{GH}\) which bisect each pair of vertical angles. \(\overline{EF}\) and \(\overline{GH}\) are perpendicular.

![Diagram of perpendicular and bisected lines]

**ANSWER:**
Sample answer: \(\overline{AB}\) and \(\overline{CD}\) intersect at \(X\) to form 2 pairs of vertical angles. The locus of points equidistant from \(\overline{AB}\) and \(\overline{CD}\) lie along \(\overline{EF}\) and \(\overline{GH}\) which bisect each pair of vertical angles. \(\overline{EF}\) and \(\overline{GH}\) are perpendicular.

43. **CHALLENGE** Suppose a line perpendicular to a pair of parallel lines intersects the lines at the points \((a, 4)\) and \((0, 6)\). If the distance between the parallel lines is \(\sqrt{5} \), find the value of \(a\) and the equations of the parallel lines.

**SOLUTION:**
Substitute the coordinates of the endpoints in the Distance Formula to find the value of \(a\).
2-10 Perpendicu"lar and Distance

\[ D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]
\[ \sqrt{5} = \sqrt{(0 - a)^2 + (6 - 4)^2} \]
\[ \sqrt{5} = \sqrt{a^2 + 2^2} \]
\[ (\sqrt{5})^2 = (\sqrt{a^2 + 2^2})^2 \]
\[ 5 = a^2 + 4 \]
\[ 5 - 4 = a^2 + 4 - 4 \]
\[ 1 = a^2 \]
\[ \sqrt{1} = \sqrt{a^2} \]
\[ \pm 1 = a \]

Let \( a = 1 \). Then the slope of the line joining the points \((1, 4)\) and \((0, 6)\) is
\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]
\[ = \frac{6 - 4}{0 - 1} \]
\[ = -2 \]
So, the slope of the line perpendicular to this line will be \( \frac{1}{2} \).

Equation of a line of slope \( \frac{1}{2} \) and that has a point \((1, 4)\) on it, is
\[ y - y_1 = m(x - x_1) \]
\[ y - 4 = \frac{1}{2}(x - 1) \]
\[ y - 4 = \frac{1}{2}x - \frac{1}{2} \]
\[ y - 4 + 4 = \frac{1}{2}x - \frac{1}{2} + 4 \]
\[ y = \frac{1}{2}x + \frac{7}{2} \]
Equation of a line of slope \( \frac{1}{2} \) and that has a point \((0, 6)\) on it, is
\[ y - y_1 = m(x - x_1) \]
\[ y - 6 = \frac{1}{2}(x - 0) \]
\[ y - 6 = \frac{1}{2}x \]
\[ y - 6 + 6 = \frac{1}{2}x + 6 \]
\[ y = \frac{1}{2}x + 6 \]

Let \( a = -1 \). Then the slope of the line joining the points \((-1, 4)\) and \((0, 6)\) is
\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]
\[ = \frac{6 - 4}{0 - (-1)} \]
\[ = 2 \]
So, the slope of the line perpendicular to this line will be \( -\frac{1}{2} \).

Equation of a line of slope \( -\frac{1}{2} \) and that has a point \((-1, 4)\) on it, is
\[ y - y_1 = m(x - x_1) \]
\[ y - 4 = -\frac{1}{2}(x - (-1)) \]
\[ y - 4 = -\frac{1}{2}x - \frac{1}{2} \]
\[ y - 4 + 4 = -\frac{1}{2}x - \frac{1}{2} + 4 \]
\[ y = -\frac{1}{2}x + \frac{7}{2} \]
Equation of a line of slope \( -\frac{1}{2} \) and that has a point \((0, 6)\) on it, is
\[ y - y_1 = m(x - x_1) \]
\[ y - 6 = -\frac{1}{2}(x - 0) \]
\[ y - 6 = -\frac{1}{2}x \]
\[ y - 6 + 6 = -\frac{1}{2}x + 6 \]
\[ y = -\frac{1}{2}x + 6 \]

Therefore, the equations of the parallel lines are
2-10 Perpendiculars and Distance

\[ y = \frac{1}{2}x + 6 \text{ and } y = \frac{1}{2}x + \frac{7}{2} \]

or

\[ y = -\frac{1}{2}x + 6 \text{ and } y = -\frac{1}{2}x + \frac{7}{2}. \]

**ANSWER:**

\[ a = \pm 1; \]

\[ y = \frac{1}{2}x + 6 \text{ and } y = \frac{1}{2}x + \frac{7}{2}. \]

\[ y = -\frac{1}{2}x + 6 \text{ and } y = -\frac{1}{2}x + \frac{7}{2}. \]

44. **REASONING** Determine whether the following statement is *sometimes, always, or never* true. Explain.

*The distance between a line and a plane can be found.*

**SOLUTION:**

The distance can only be found if the line is parallel to the plane. So, the statement is *sometimes* true. If the line is not parallel to the plane, the distance from the plane to one point on the line is different than the distance to a different point on the line.

**ANSWER:**

Sometimes; the distance can only be found if the line is parallel to the plane.

45. **OPEN-ENDED** Draw an irregular convex pentagon using a straightedge.

**a.** Use a compass and straightedge to construct a line between one vertex and a side opposite the vertex.

**b.** Use measurement to justify that the line constructed is perpendicular to the chosen side.

**c.** Use mathematics to justify this conclusion.

**SOLUTION:**

**a.** Sample answer:

**b.** Sample answer: Using a protractor, the measurement of the constructed angle is equal to 90. So, the line constructed from vertex \( P \) is perpendicular to the nonadjacent chosen side.

**c.** Sample answer: The same compass setting was used to construct points \( A \) and \( B \). Then the same compass setting was used to construct the perpendicular line to the chosen side. Since the compass setting was equidistant in both steps a perpendicular line was constructed.
46. SENSE-MAKING Rewrite Theorem 2.25 in terms of two planes that are equidistant from a third plane. Sketch an example.

**SOLUTION:**
If two planes are each equidistant form a third plane, then the two planes are parallel to each other.

**ANSWER:**
If two planes are each equidistant form a third plane, then the two planes are parallel to each other.

47. WRITING IN MATH Describe how you can use the Distance Formula to verify that two lines are parallel.

**SOLUTION:**
Since parallel lines are equidistant everywhere, find the distance between a pair of points which form a line segment perpendicular to the given lines.

Determine the slope of each line. Then select a point on the first line. Find the equation of the line perpendicular to the first line through the given point.

Then solve a system of equation using the perpendicular line and the second parallel line. This will give you a second point. Use the distance formula to find the distance between the two points.

Select a second point on the first parallel. Repeat the process of finding a point on the 2nd parallel line. When you use the distance formula to find the distance between the two points, it should be the same as for the first pair of points.
So, the distance should be the same for any two points on perpendicular lines.

**ANSWER:**
Since parallel lines are equidistant everywhere, find the distance between a pair of points, which form a line segment perpendicular to the given lines. The Distance Formula is used to determine the distance between the pair of intersection points. Repeat this process for a second pair of points on the lines. Use the Distance Formula to find the distance between these points. This distance should be the same as the distance between the first pair of points.
48. **MULTI-STEP** The diagram shows line $XY$ and point $P$.

**a.** Find the coordinates $a$ and $b$.

**b.** Find the distance between point $P$ and line $XY$.

**c.** Find the coordinates of the point $Q$ where $XY$ is the perpendicular bisector of $PQ$.

**SOLUTION:**

**a.** Find the coordinates $a$ and $b$.

The coordinates of the point on the line that is closest to $P$ is on a line perpendicular with slope -1. The line is $y = -x + 5$, so the coordinates are at (-1, 6). Since that is the point on both $y = -x + 5$ and $y = x + 7$.

**b.** Find the distance between point $P$ and line $XY$.

$$
\sqrt{(3 - (-1))^2 + (2 - 6)^2} = \sqrt{16 + 16} = \sqrt{32} = 4\sqrt{2}
$$

**c.** Find the coordinates of the point $Q$ where $XY$ is the perpendicular bisector of $PQ$.

This is the point that is the same distance away from the line, but in the opposite direction, which is 4 units left and 4 units up from (-1, 6) or (-5, 10)

**ANSWER:**

**a.** (-1, 6). Since that is the point on both $y = -x + 5$ and $y = x + 7$.

**b.** $4\sqrt{2}$

**c.** (-5, 10)

49. In the diagram, the dashed line through (0, -2) is perpendicular to line $q$.

What are the coordinates of the intersection of the dashed line and line $p$?

**A** (8.0, 2.0)
**B** (4.4, 3.2)
**C** (3.2, 4.4)
**D** (3.2, -3.6)

**SOLUTION:**

Let equation 1 be $y = -0.5x - 2$ and equation 2 be $y = -0.5x + 6$.

The slope of the line $y = -0.5x - 2$ is -0.5. A line perpendicular will have the slope of 2.

Use the slope and the point (0, -2) to write the equation of the line. Let $(x_1, y_1) = (0, -2)$.

$$
y - y_1 = m(x - x_1) \quad \text{Point-Slope form}$$

$$
y - (-2) = 2(x - 0) \quad \text{Substitution}$$

$$
y + 2 = 2x \quad \text{Equation 3}$$

Solve the system of equations for equation 2 and 3 to determine the point of intersection.

The left sides of the equations are the same. So, equate the right sides and solve for $x$.

$$
-0.5x + 6 = 2x + 2 \quad \text{Equation 2 = Equation 3}
$$

Use the value of $x$ to find the value of $y$.

$$y = 2x - 2 \quad \text{Equation 2}$$

$$= 2(3.2) - 2$$

$$= 4.4 \quad \text{y-coord of pt. of intersection}$$

So, the point of intersection is (3.2, 4.4). Thus, the correct choice is C.

**ANSWER:**

C
50. What is the shortest distance from the point \((4, 9)\) to the line \(y = -x + 6\) to the nearest tenth unit?

**SOLUTION:**
The slope of the line \(y = -x + 6\) is -1. A line perpendicular to the line will have slope of 1.

Use the slope and the points to write the equation of the line. Let \((x_1, y_1) = (4, 9)\).

\[
\begin{align*}
y - y_1 &= m(x - x_1) & \text{Point-Slope form} \\
y - 9 &= 1(x - 4) & \text{Substitution} \\
y - 9 &= x - 4 \\
y + 9 - 9 &= x - 4 + 9 \\
y &= x + 5 & \text{Equation 2}
\end{align*}
\]

Solve the system of equations to determine the point of intersection.

\[
\begin{align*}
x + 5 &= -x + 6 & \text{Equation 1} \\
x + x + 5 &= -x + x + 6 \\
2x + 5 &= 6 \\
2x - 5 &= 6 - 5 \\
2x &= 1 \\
2 &= 2 \\
x &= \frac{1}{2} & \text{x-coord of pt of intersection}
\end{align*}
\]

Use the value of \(x\) to find the value of \(y\).

\[
y = -x + 6 & \quad \text{Equation 1} \\
= -\frac{1}{2} + 6 & \quad \text{Substitution} \\
= 5.5 & \quad \text{y-coord of pt of intersection}
\]

So, the point of intersection is \((0.5, 5.5)\).

Use the Distance Formula to find the distance between the points \((4, 9)\) and \((0.5, 5.5)\). Let \((x_1, y_1) = (0.5, 5.5)\) and \((x_2, y_2) = (4, 9)\).

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
= \sqrt{(4 - 0.5)^2 + (9 - 5.5)^2} \\
= \sqrt{12.25 + 12.25} \\
= \sqrt{24.5}
\]

Therefore, the shortest distance between the line and the point is about 4.9 units.

**ANSWER:**
4.9

51. The manager of a park wants to find the distance from the tree at point \(T\) to the path \(\overline{PH}\). What are the first two construction steps to find that distance?

**A** Measure the distance \(TP\), and then measure the distance \(TH\).

**B** Use an arc to find two points \(X\) and \(Y\) on line \(\overline{PH}\) that are the same distance from \(T\). Then find another point that is equidistant from points \(X\) and \(Y\).

**C** Draw a line segment from \(T\) to \(H\), and then bisect segment \(TH\).

**D** Find the midpoint \(M\) of \(\overline{PH}\), and then draw \(\overline{TM}\).

**E** Construct the line parallel to \(\overline{PH}\) through \(H\), and then use a ruler to measure the distance.

**SOLUTION:**

![Diagram](image)

Place the compass on point \(T\), and draw an arc that intersects \(\overline{PH}\) twice. Label the points \(X\) and \(Y\).

Then place the compass on point \(X\) and \(Y\) using the same setting and draw arcs to find another point that is equidistant from points \(X\) and \(Y\). Label this point \(M\). Thus, choice B is correct.

**ANSWER:**
B
2-10 Perpendiculars and Distance

52. Given \( y = 3x + 2 \), write the following equations:

   a. an equation parallel to the given line

   b. an equation perpendicular to the given line with the same \( y \)-intercept

   c. an equation perpendicular to the given line with a different \( y \)-intercept

**SOLUTION:**

a. an equation parallel to \( y = 3x + 2 \) is \( y = 3x + 5 \), because they have the same slope.

b. an equation perpendicular to \( y = 3x + 2 \) with the same \( y \)-intercept is \( y = -\frac{1}{3}x + 2 \), because their slopes have a product of -1, and they have the same value of \( b \) in the \( y = mx + b \) form.

C. an equation perpendicular to \( y = 3x + 2 \) with a different \( y \)-intercept is \( y = -\frac{1}{3}x + 5 \), because their slopes have a product of -1, and they have a different value of \( b \) in the \( y = mx + b \) form.

**ANSWER:**

a. Sample answer: \( y = 3x + 5 \)

b. Sample answer: \( y = -\frac{1}{3}x + 2 \)

c. Sample answer: \( y = -\frac{1}{3}x + 5 \)

53. Graph the lines \( y = 2x + 4 \) and \( y = -\frac{1}{2}x - \frac{7}{2} \). What is the point of intersection of the two lines?

A (5, -6)
B (-6, -5)
C (5, 6)
D (6, 5)

**SOLUTION:**

Graph the lines on the same coordinate plane and find their intersection.

**ANSWER:**

A