Classify the relationship between each pair of angles as alternate interior, alternate exterior, corresponding, or consecutive interior angles.

1. \( \angle 1 \) and \( \angle 8 \\
SOLUTION:
Exterior angles that are non adjacent and lie on opposite sides of the transversal are alternate exterior angles.

2. \( \angle 2 \) and \( \angle 4 \\
SOLUTION:
Interior angles that lie on the same side of the transversal are corresponding angles.

3. \( \angle 3 \) and \( \angle 6 \\
SOLUTION:
Interior angles that are non adjacent and lie on opposite sides of the transversal are alternate interior angles.

4. \( \angle 6 \) and \( \angle 7 \\
SOLUTION:
Interior angles that lie on the same side of the transversal are consecutive interior angles.

In the figure, \( m\angle 1 = 94 \). Find the measure of each angle. Tell which postulate(s) or theorem(s) you used.

5. \( \angle 3 \\
SOLUTION:
In the figure, angles 1 and 3 are corresponding angles. Use the Corresponding Angles Postulate. If two parallel lines are cut by a transversal, then each pair of corresponding angles are congruent.

\[
\angle 3 \cong \angle 1 \\
\text{Corresponding Angles Postulate} \\
m\angle 3 = m\angle 1 \\
\text{Definition of congruent angles} \\
m\angle 3 = 94 \\
\text{Substitution.}
\]

6. \( \angle 5 \\
SOLUTION:
In the figure, angles 1 and 5 are alternate exterior angles.

Use the Alternate Exterior Angles Theorem. If two parallel lines are cut by a transversal, then each pair of alternate interior angles is congruent.

\[
\angle 5 \cong \angle 1 \\
\text{Alternate Exterior Angles Theorem} \\
m\angle 5 = m\angle 1 \\
\text{Definition of congruent angles} \\
m\angle 5 = 94 \\
\text{Substitution.}
\]

7. \( \angle 4 \\
SOLUTION:
In the figure, angles 1 and 3 are corresponding angles. Use the Corresponding Angles Postulate: If two parallel lines are cut by a transversal, then each pair of corresponding angles is congruent.

\[
\angle 3 \cong \angle 1 \\
\text{Corresponding Angles Postulate} \\
m\angle 3 = m\angle 1 \\
\text{Definition of congruent angles} \\
m\angle 3 = 94 \\
\text{Substitution.}
\]

\[
\angle 3 + \angle 4 = 180^\circ \\
\text{Def. of supplementary angles} \\
m\angle 3 + m\angle 4 = 180 \\
\text{Def. of congruent angles} \\
94 + m\angle 4 = 180 \\
\text{Substitution.} \\
94 - 94 + m\angle 4 = 180 - 94 \\
m\angle 4 = 86 \\
\text{Simplify.}
\]
In the figure, $m \angle 4 = 101$. Find the measure of each angle. Tell which postulate(s) or theorem(s) you used.

8. $\angle 6$

**SOLUTION:**
In the figure, angles 4 and 6 are alternate interior angles. Use the Alternate interior Angles Theorem.

If two parallel lines are cut by a transversal, then each pair of alternate interior angles is congruent.

\[
\angle 4 \cong \angle 6 \quad \text{Alternate Interior Angles Theorem}
\]
\[
m_\angle 4 = m_\angle 6 \quad \text{Definition of congruent angles}
\]
\[
101 = m_\angle 6 \quad \text{Substitution}
\]

9. $\angle 7$

**SOLUTION:**
In the figure, angles 4 and 5 are consecutive interior angles.

\[
\angle 4 + \angle 5 = 180^\circ \quad \text{Def. of supplementary angles}
\]
\[
m_\angle 4 + m_\angle 5 = 180
\]
\[
101 + m_\angle 5 = 180 \quad \text{Substitution}
\]
\[
101 + m_\angle 5 = 180 - 101 \quad \text{Subtract 101 from each side.}
\]
\[
m_\angle 5 = 79 \quad \text{Simplify}
\]

In the figure, angles 5 and 7 are vertical angles.

\[
\angle 5 \cong \angle 7 \quad \text{Vertical Angles}
\]
\[
m_\angle 5 = m_\angle 7 \quad \text{Definition of congruent angles}
\]
\[
79 = m_\angle 7 \quad \text{Substitution}
\]

10. $\angle 5$

**SOLUTION:**
In the figure, angles 4 and 5 are consecutive interior angles.

\[
\angle 4 + \angle 5 = 180^\circ \quad \text{Consecutive Interior Angles Theorem}
\]
\[
m_\angle 4 + m_\angle 5 = 180
\]
\[
101 + m_\angle 5 = 180 \quad \text{Substitution}
\]
\[
101 - 101 + m_\angle 5 = 180 - 101 \quad \text{Subtract 101 from each side.}
\]
\[
m_\angle 5 = 79 \quad \text{Simplify}
\]

11. **ROADS**

In the diagram, the guard rail is parallel to the surface of the roadway and the vertical supports are parallel to each other. Find the measures of angles 2, 3, and 4.

**SOLUTION:**
Use the Alternate Interior Angles Theorem, Definition of Supplementary Angles and Corresponding Angles Postulate to find $m _\angle 4$. 

\[
\angle 2 = 93^\circ \quad \text{Alternate Interior Angles Theorem}
\]
\[
m_\angle 2 = 93
\]
\[
\angle 3 + 93^\circ = 180^\circ \quad \text{Definition of supplementary angles}
\]
\[
m_\angle 3 + 93 = 180
\]
\[
m_\angle 3 + 93 - 93 = 180 - 93 \quad \text{Subtract 93 from each side.}
\]
\[
m_\angle 3 = 87 \quad \text{Simplify}
\]

\[
\angle 4 = 87^\circ \quad \text{Corresponding Angles Postulate}
\]
\[
m_\angle 4 = 87
\]
\[
\text{So, } m_\angle 4 = 87.
\]

Find the value of the variable(s) in each figure. Explain your reasoning.

12. $55^\circ$

**SOLUTION:**
Use the definition of supplementary angles to find $m _\angle x$. Then use the Alternate Interior Angles Theorem to find $m _\angle y$.

\[
m_\angle x + 55 = 180^\circ \quad \text{Def. of supplementary angles}
\]
\[
m_\angle x + 55 - 55 = 180 - 55
\]
\[
m_\angle x = 125 \quad \text{Simplify}
\]

\[
\angle x \cong \angle y \quad \text{Alternate Interior Angles Theorem}
\]
\[
m_\angle x = m_\angle y \quad \text{Definition of congruent angles}
\]
\[
125 = m_\angle y \quad \text{Substitution}
\]
13. **SOLUTION:**

Use the Alternate Exterior Angles Theorem to find \( x \).

\[
104 = x - 10 \quad \text{Alternate Exterior Angles Theorem}
\]

\[
104 + 10 = x - 10 + 10 \quad \text{Add 10 to each side.}
\]

\[
x = 114 \quad \text{Simplify.}
\]

14. **SOLUTION:**

Use the Alternate Interior Angles Theorem to find \( x \).

\[
x + 55 = 2x - 15 \quad \text{Alternate Interior Angles Theorem}
\]

\[
x - 2x + 55 = 2x - 2x - 15 \quad \text{Subtract } 2x \text{ from each side.}
\]

\[-x + 55 = -15 \quad \text{Simplify.}
\]

\[-x + 55 - 55 = -15 - 55 \quad \text{Subtract } 55 \text{ from each side.}
\]

\[-x = -70 \quad \text{Simplify.}
\]

\[-1(-x) = -1(-70) \quad \text{Multiply each side by } -1.
\]

\[x = 70 \quad \text{Simplify.}
\]

**PRECISION** Classify the relationship between each pair of angles as *alternate interior*, *alternate exterior*, *corresponding*, or *consecutive interior angles*.

15. \( \angle 4 \) and \( \angle 9 \)

**SOLUTION:**

The transversal connecting \( \angle 4 \) and \( \angle 9 \) is line \( s \). One interior (\( \angle 9 \)) and one exterior (\( \angle 4 \)) (non adjacent) angles that lie on the same side of the transversal are corresponding angles.

16. \( \angle 5 \) and \( \angle 7 \)

**SOLUTION:**

The transversal connecting \( \angle 5 \) and \( \angle 7 \) is line \( r \). Interior angles that lie on the same side of the transversal are consecutive interior angles.

17. \( \angle 3 \) and \( \angle 5 \)

**SOLUTION:**

The transversal connecting \( \angle 3 \) and \( \angle 5 \) is line \( t \). Interior angles that are non adjacent and lie on opposite sides of the transversal are alternate interior angles.

18. \( \angle 10 \) and \( \angle 11 \)

**SOLUTION:**

The transversal connecting \( \angle 10 \) and \( \angle 11 \) is line \( v \). One interior (\( \angle 11 \)) and one exterior (\( \angle 10 \)) (non adjacent) angles that lie on the same side of the transversal are corresponding angles.

19. \( \angle 1 \) and \( \angle 6 \)

**SOLUTION:**

The transversal connecting \( \angle 1 \) and \( \angle 6 \) is line \( t \). Exterior angles that are non adjacent and lie on opposite sides of the transversal are alternate exterior angles.

20. \( \angle 6 \) and \( \angle 8 \)

**SOLUTION:**

The transversal connecting \( \angle 6 \) and \( \angle 8 \) is line \( s \). Interior angles that lie on the same side of the transversal are consecutive interior angles.

21. \( \angle 2 \) and \( \angle 3 \)

**SOLUTION:**

The transversal connecting \( \angle 2 \) and \( \angle 3 \) is line \( t \). Interior angles that lie on the same side of the transversal are consecutive interior angles.

22. \( \angle 9 \) and \( \angle 10 \)

**SOLUTION:**

The transversal connecting \( \angle 9 \) and \( \angle 10 \) is line \( v \). Exterior angles that are non adjacent and lie on opposite sides of the transversal are alternate exterior angles.
23. \( \angle 4 \) and \( \angle 11 \)

**SOLUTION:**
The transversal connecting \( \angle 4 \) and \( \angle 11 \) is line \( s \).
Exterior angles that are non adjacent and lie on opposite sides of the transversal are alternate exterior angles.

24. \( \angle 7 \) and \( \angle 11 \)

**SOLUTION:**
The transversal connecting \( \angle 7 \) and \( \angle 11 \) is line \( v \).
Interior angles that are non adjacent and lie on opposite sides of the transversal are alternate interior angles.

In the figure, \( m \angle 11 = 62 \) and \( m \angle 14 = 38 \). Find the measure of each angle. Tell which postulate (s) or theorem(s) you used.

25. \( \angle 4 \)

**SOLUTION:**
In the figure, angles 4 and 11 are corresponding angles.
\[ \angle 4 \cong \angle 11 \quad \text{Corresponding Angles Postulate} \]
\[ m\angle 4 = m\angle 11 \quad \text{Definition of congruent angles} \]
\[ m\angle 4 = 62 \quad \text{Substitution} \]

26. \( \angle 3 \)

**SOLUTION:**
In the figure, angles 4 and 11 are corresponding angles and angles 3 and 4 are vertical angles.
\[ \angle 4 \cong \angle 11 \quad \text{Corresponding Angles Postulate} \]
\[ m\angle 4 = m\angle 11 \quad \text{Definition of congruent angles} \]
\[ m\angle 4 = 62 \quad \text{Substitution} \]
\[ \angle 3 \cong \angle 4 \quad \text{Vertical Angles} \]
\[ m\angle 3 = m\angle 4 \quad \text{Definition of congruent angles} \]
\[ m\angle 3 = 62 \quad \text{Substitution} \]

27. \( \angle 12 \)

**SOLUTION:**
In the figure, angles 12 and 11 are supplementary angles.
\[ m\angle 11 + m\angle 12 = 180^\circ \quad \text{Definition of supplementary angles} \]
\[ 66 + m\angle 12 = 180 \quad \text{Substitution} \]
\[ 180 - 66 - m\angle 12 = 180 - 66 \]
\[ m\angle 12 = 114 \quad \text{Subtract 66 from each side.} \]

28. \( \angle 8 \)

**SOLUTION:**
In the figure, angles 8 and 11 are vertical angles.
\[ \angle 8 \cong \angle 11 \quad \text{Vertical Angles} \]
\[ m\angle 8 = m\angle 11 \quad \text{Definition of congruent angles} \]
\[ m\angle 8 = 62 \quad \text{Substitution} \]

29. \( \angle 6 \)

**SOLUTION:**
In the figure, angles 14 and 6 are corresponding angles.
\[ \angle 6 \cong \angle 14 \quad \text{Corresponding Angles Postulate} \]
\[ m\angle 6 = m\angle 14 \quad \text{Definition of congruent angles} \]
\[ m\angle 6 = 38 \quad \text{Substitution} \]

30. \( \angle 2 \)

**SOLUTION:**
The angles \( \angle 1 \) and \( \angle 14 \) are alternate exterior angles and so are congruent. and angles \( \angle 3 \) and \( \angle 11 \) are alternate exterior angles and so are congruent. By Supplementary Theorem, \( m\angle 1 + m\angle 2 + m\angle 3 = 180 \).
\[ \angle 1 \cong \angle 14 \quad \text{Alternate Exterior Angles Theorem} \]
\[ m\angle 1 = m\angle 14 \quad \text{Definition of congruent angles} \]
\[ m\angle 1 = 38 \quad \text{Substitution} \]
\[ \angle 3 \cong \angle 11 \quad \text{Alternate Exterior Angles Theorem} \]
\[ m\angle 3 = m\angle 11 \quad \text{Definition of congruent angles} \]
\[ m\angle 3 = 62 \quad \text{Substitution} \]
\[ m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ \quad \text{Ref. of supplementary angles} \]
\[ m\angle 1 + \angle 2 + m\angle 3 = 180 \]
\[ m\angle 1 + 66 + m\angle 3 = 180 \quad \text{Substitution} \]
\[ 38 + m\angle 2 + 62 = 180 \quad \text{Simplify} \]
\[ 62 + m\angle 2 = 180 \]
\[ 180 - 62 + m\angle 2 = 180 - 62 \]
\[ m\angle 2 = 180 - 62 \quad \text{Subtract 62 from each side.} \]
\[ m\angle 2 = 118 \quad \text{Simplify} \]
2-7 Parallel Lines and Transversals

31. \( \angle 10 \)

**SOLUTION:**
In the figure, angles 14 and 10 are supplementary angles.
\[
\angle 14 + \angle 10 = 180^\circ \\
m\angle 14 + m\angle 10 = 180 \\
38 + m\angle 10 = 180 \\
38 - 38 + m\angle 10 = 180 - 38 \\
m\angle 10 = 142
\]

32. \( \angle 5 \)

**SOLUTION:**
Use definition of supplementary angles, Corresponding Angles Postulate and the Alternate Interior Angles Theorem.
\[
\angle 11 + \angle 7 = 180^\circ \\
m\angle 11 + m\angle 7 = 180 \\
62 + m\angle 7 = 180 \\
62 - 62 + m\angle 7 = 180 - 62 \\
m\angle 7 = 118
\]

\[
\angle 6 \cong \angle 14 \\
m\angle 6 = m\angle 14 \\
m\angle 6 = 38
\]

33. \( \angle 1 \)

**SOLUTION:**
In the figure, angles 1 and 14 are alternate exterior angles.
\[
\angle 1 \cong \angle 14 \\
m\angle 1 = m\angle 14 \\
m\angle 1 = 38
\]

34. **Find the value of the variable(s) in each figure. Explain your reasoning.**

\[x + 10^\circ = 114^\circ \]

**SOLUTION:**
Use Corresponding Angles Postulate and definition of supplementary angles to find \(x\).
\[
m\angle x + (x + 12)^\circ = 180 \\
n114 + x + 12 = 180 \\
126 + x = 180 \\
x = 54
\]

35. **Find the value of the variable(s) in each figure. Explain your reasoning.**

\[3(\angle x - 15)^\circ = 105^\circ \]

**SOLUTION:**
Use the Corresponding Angles Postulate and Supplement Theorem to find \(x\) and \(y\).
\[
3(\angle x - 15)^\circ = 105^\circ \\
\angle x - 15 = 35 \\
3\angle x = 120 \\
x = 40
\]

\[3(\angle y + 25)^\circ = 160^\circ \]

**SOLUTION:**
Use the Corresponding Angles Postulate and Supplement Theorem to find \(x\) and \(y\).
\[
3\angle y + 25 = 160 \\
\angle y + 25 = 50 \\
\angle y = 25
\]
36. **SOLUTION:**
Use the Vertical Angle Theorem and Consecutive Interior Angles Theorem to find $x$.

\[
(2x)^\circ + 54^\circ = 180^\circ \\
2x + 54 = 180 \\
2x = 126 \\
x = 63
\]

37. **SOLUTION:**
Use the Consecutive Interior Angles Theorem to find $x$ and $y$.

\[
(2x)^\circ + 96^\circ = 180^\circ \\
2x + 96 = 180 \\
2x = 84 \\
x = 42
\]

\[
94^\circ + (3y + 44)^\circ = 180^\circ \\
94 + 3y + 44 = 180 \\
3y + 138 = 180 \\
3y = 42 \\
\frac{3y}{3} = \frac{42}{3} \\
y = 14
\]

38. **SOLUTION:**
Use the Alternate Interior Angles Theorem and Consecutive Interior Angles Theorem to find $x$ and $y$.

\[
(2x)^\circ = 108^\circ \\
2x = 108 \\
2 \cdot \frac{x}{2} = \frac{108}{2} \\
x = 54
\]

39. **SOLUTION:**
Use the Consecutive Interior Angles Theorem and definition of supplementary angles to find $x$ and $y$.

\[
x^\circ + 120^\circ = 180^\circ \\
x + 120 = 180 \\
x = 60
\]

\[
(3x - 70)^\circ + (3y + 40)^\circ = 180^\circ \\
3x - 70 + 3y + 40 = 180 \\
3x + 3y = 180 \\
150 + 3y = 180 \\
3y = 30 \\
\frac{3y}{3} = \frac{30}{3} \\
y = 10
\]
40. **PROOF** Copy and complete the proof of Theorem 2.15.

Given: \( m \parallel n; \ell \) is a transversal.
Prove: \( \angle 1 \) and \( \angle 2 \) are supplementary; \( \angle 3 \) and \( \angle 4 \) are supplementary.

**Proof:**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ?</td>
<td>a. Given</td>
</tr>
<tr>
<td>b. ( \angle 1 ) and ( \angle 3 ) form a linear pair; ( \angle 2 ) and ( \angle 4 ) form a linear pair.</td>
<td>b. ?</td>
</tr>
<tr>
<td>c. ?</td>
<td>c. If two angles form a linear pair, then they are supplementary.</td>
</tr>
<tr>
<td>d. ( \angle 1 \cong \angle 4 ), ( \angle 2 \cong \angle 3 )</td>
<td>d. ?</td>
</tr>
<tr>
<td>e. ( m\angle 1 = m\angle 4, m\angle 2 = m\angle 3 )</td>
<td>e. Definition of Congruence</td>
</tr>
<tr>
<td>f. ?</td>
<td>f. ?</td>
</tr>
</tbody>
</table>

**SOLUTION:**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ( m \parallel n; \ell ) is a transversal.</td>
<td>a. Given</td>
</tr>
<tr>
<td>b. ( \angle 1 ) and ( \angle 3 ) form a linear pair; ( \angle 2 ) and ( \angle 4 ) form a linear pair.</td>
<td>b. Def. of linear pair</td>
</tr>
<tr>
<td>c. ( \angle 1 ) and ( \angle 3 ) are supplementary; ( \angle 2 ) and ( \angle 4 ) are supplementary.</td>
<td>c. If two angles form a linear pair, then they are supplementary.</td>
</tr>
<tr>
<td>d. ( \angle 1 \cong \angle 4 ), ( \angle 2 \cong \angle 3 )</td>
<td>d. Alt. Int. ( \angle )'s Theorem</td>
</tr>
<tr>
<td>e. ( m\angle 1 = m\angle 4, m\angle 2 = m\angle 3 )</td>
<td>e. Definition of Congruence</td>
</tr>
<tr>
<td>f. ( \angle 1 ) and ( \angle 2 ) are supp; ( \angle 3 ) and ( \angle 4 ) are supp.</td>
<td>f. Substitution</td>
</tr>
</tbody>
</table>

41. \( \angle 1 \) and \( \angle 8 \)

**SOLUTION:**

\( \angle 1 \) and \( \angle 8 \) are Alternate interior angles. Therefore \( \angle 1 \) and \( \angle 8 \) are congruent.
42. \( \angle 1 \) and \( \angle 5 \)

**SOLUTION:**
\( \angle 1 \) and \( \angle 5 \) are Corresponding angles. Therefore, they are congruent.

43. \( \angle 3 \) and \( \angle 6 \)

**SOLUTION:**
\( \angle 3 \) and \( \angle 6 \) are Vertical angles. Therefore Vertical angles are congruent.

44. \( \angle 1 \) and \( \angle 2 \)

**SOLUTION:**
All vertical and horizontal lines are perpendicular at their point of intersection. By definition of perpendicular, they form right angles. \( \angle 1 \) and \( \angle 2 \) are adjacent angles. By the Angle Addition Postulate, \( m\angle 1 + m\angle 2 = 90 \). Since the sum of the two angles is 90, \( \angle 1 \) and \( \angle 2 \) are complementary angles.
45. **CONSTRUCT ARGUMENTS** Write a two-column proof of the Alternate Exterior Angles Theorem.

**SOLUTION:**

Given: $\ell \parallel m$

Prove: $\angle 1 \cong \angle 8$

$\angle 2 \cong \angle 7$

![Diagram](image)

Proof:

<table>
<thead>
<tr>
<th>Statements (Reasons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\ell \parallel m$ (Given)</td>
</tr>
<tr>
<td>2. $\angle 1 \cong \angle 5$, $\angle 2 \cong \angle 6$ (Corr. $\angle$s Post.)</td>
</tr>
<tr>
<td>3. $\angle 8 \cong \angle 5$, $\angle 7 \cong \angle 6$ (Vertical $\angle$ Thm.)</td>
</tr>
<tr>
<td>4. $\angle 8 \cong \angle 1$, $\angle 7 \cong \angle 2$ (Trans. Prop.)</td>
</tr>
</tbody>
</table>

**BRIDGES**

Refer to the diagram of the double-decker Michigan Avenue Bridge in Chicago, Illinois. The top and bottom beams and its diagonal braces are parallel.

a. How are the measures of the odd-numbered angles related? Explain.

b. How are the measures of the even-numbered angles related? Explain.

c. How are any pair of angles in which one is odd and the other is even related?

d. What geometric term(s) can be used to relate the two roadways contained by the bridge?

**SOLUTION:**

a. The top and bottom levels of the bridge are parallel, so the lines formed by the edge of each level are parallel, and by using the diagonal braces as transversals and the Alternate Interior Angles Theorem $\angle 1 \cong \angle 3$, $\angle 5 \cong \angle 7$, $\angle 9 \cong \angle 11$, and $\angle 13 \cong \angle 15$.

The diagonal braces are parallel, so by using the vertical braces as transversals and the Alternate Interior Angles Theorem $\angle 4 \cong \angle 6$, $\angle 8 \cong \angle 10$, and $\angle 12 \cong \angle 14$.

Since the vertical braces are perpendicular to the levels of the bridge, $\angle 3$ and $\angle 4$, $\angle 5$ and $\angle 6$, $\angle 7$ and $\angle 8$, $\angle 9$ and $\angle 10$, $\angle 11$ and $\angle 12$, and $\angle 13$ and $\angle 14$ are pairs of complementary angles.

By the Congruent Complements Theorem, $\angle 3 \cong \angle 5$, $\angle 7 \cong \angle 9$, and $\angle 11 \cong \angle 13$. So, $\angle 1 \cong \angle 3 \cong \angle 5$, $\angle 7 \cong \angle 9$, $\angle 11 \cong \angle 13$ by the Transitive Property of Congruence. So, all of the odd numbered angles are alternate interior angles related by the diagonal transversals or are complements of even numbered alternate interior angles related by the vertical transversals. Therefore, they are all congruent.

b. All the vertical braces are parallel since all vertical lines are parallel. Using the diagonal braces as transversals to the vertical braces and the Alternate Interior Angles Theorem, $\angle 2 \cong \angle 4$, $\angle 6 \cong \angle 8$, $\angle 10 \cong \angle 12$, and $\angle 14 \cong \angle 16$.

Using the vertical braces as transversals between the diagonal braces and the Alternate Interior Angles Theorem, $\angle 4 \cong \angle 6$, $\angle 8 \cong \angle 10$, and $\angle 12 \cong \angle 14$. So, $\angle 2 \cong \angle 4 \cong \angle 6 \cong \angle 8 \cong \angle 10 \cong \angle 12 \cong \angle 14 \cong \angle 16$ by the Transitive Property of Congruence.

All of the even numbered angles are alternate interior angles related by either the diagonal transversals or the vertical transversals. Therefore, they are all congruent.

c. Complementary; since the vertical supports and the horizontal supports are perpendicular, angle pairs like angle 1 and angle 2 must be complementary.

Since all of the odd numbered angles are congruent and all of the even numbered angles are congruent, any pair of angles that has one odd and one even number will be complementary.

d. Since the two levels (or surfaces) of the bridge are parallel, the geometric term that best represents the two roadways contained by the bridge is parallel planes.
47. **PROOF** In a plane, prove that if a line is perpendicular to one of two parallel lines, then it is perpendicular to the other.

**SOLUTION:**
Given: \( m \parallel n, t \perp m \)
Prove: \( t \perp n \)

Proof:

**Statements (Reasons)**
1. \( m \parallel n, t \perp m \) (Given)
2. Angle 1 is a right angle. (Def. of \( \perp \))
3. \( m \angle 1 = 90 \) (Def. of \( \angle s \))
4. \( \angle 1 \cong \angle 2 \) (Corr. \( \angle s \) Post.)
5. \( m \angle 1 = m \angle 2 \) (Def. of \( \cong \angle s \))
6. \( m \angle 2 = 90 \) (Subs.)
7. \( \angle 2 \) is a right angle. (Def. of \( \angle s \))
8. \( t \perp n \) (Def. of \( \perp \) lines)

**TOOLS** Find \( x \).

48. **SOLUTION:**

**Draw an auxiliary line to construct a triangle.**

Then label the angles \( a^\circ, b^\circ, \) and \( c^\circ \). By finding the measures for angles \( a \) and \( b \), we can use the Triangle Angle Sum theorem to find angle \( c \). Angles \( c \) and \( x \) are vertical angles.

Use the definition of supplementary angles to find \( a \).

\[
72^\circ + a^\circ = 180^\circ \quad \text{Def. of supplementary angles} \\
72 + a = 180 \quad \text{Def. of congruent angles} \\
72 - 72 + a = 180 - 72 \quad \text{Subtract 72 from each side.} \\
a = 108 \quad \text{Simplify}
\]

Find angle \( b \).

\( b^\circ \cong 50 \quad \text{Corresponding Angles Theorem} \)

\( b = 50 \quad \text{Definition of congruent angles} \)

Find angle \( c \).

\[
\begin{align*}
\alpha^\circ + b^\circ + c^\circ & \equiv 180^\circ \quad \text{Triangle Angle Sum Theorem} \\
\alpha + b + c & = 180 \quad \text{Definition of congruent angles} \\
108 + 50 + c & = 180 \quad \text{Substitution} \\
158 + c & = 180 \quad \text{Simplify} \\
158 - 158 + c & = 180 - 158 \quad \text{Subtract 158 from each side.} \\
c & = 22 \quad \text{Simplify}
\end{align*}
\]

Find angle \( x \).

\( c^\circ \cong x^\circ \quad \text{Vertical Angles} \)

\( c = x \quad \text{Def. of congruent angles} \)

\( 22 = x \quad \text{Substitution} \)

So, \( x = 22 \).
49. **SOLUTION:**
Draw an auxiliary line to construct a triangle. By creating a triangle, we can use the Triangle Angle Sum Theorem and definition of supplementary angles to find x. Label the angles.

First find angle a.

\[ a + 125^\circ = 180^\circ \quad \text{Def. of supplementary angles} \]
\[ a + 125 = 180 \quad \text{Def. of congruent angles} \]
\[ a + 125 - 125 = 180 - 125 \quad \text{Subtract 125 from each side.} \]
\[ a = 55 \quad \text{Simplify.} \]

Find angle b.

\[ a^\circ = b^\circ \quad \text{Alternate Interior Angles Theorem} \]
\[ a = b \quad \text{Definition of congruent angles} \]
\[ 55 = b \quad \text{Substitution.} \]

Find angle c.

\[ b^\circ + c^\circ + d^\circ = 180^\circ \quad \text{Def. of supplementary angles} \]
\[ c + 105 = 180 \quad \text{Def. of congruent angles} \]
\[ c + 105 - 105 = 180 - 105 \quad \text{Subtract 105 from each side.} \]
\[ c = 75 \quad \text{Simplify.} \]

Find angle d.

\[ b^\circ + c^\circ + d^\circ = 180^\circ \quad \text{Triangle Angle Sum Theorem} \]
\[ b + c + d = 180 \quad \text{Def. of congruent angles} \]
\[ 55 + 75 + d = 180 \quad \text{Substitution.} \]
\[ 130 + d = 180 \quad \text{Simplify.} \]
\[ 130 - 130 + d = 180 - 130 \quad \text{Subtract 130 from each side.} \]
\[ d = 50 \quad \text{Simplify.} \]

Find angle x.

\[ x^\circ + c^\circ = 180^\circ \quad \text{Def. of supplementary angles} \]
\[ x + 50 = 180 \quad \text{Def. of congruent angles} \]
\[ x + 50 - 50 = 180 - 50 \quad \text{Subtract 50 from each side.} \]
\[ x = 130 \quad \text{Simplify.} \]

So \( x = 130^\circ \).

50. **PROBABILITY** Suppose you were to pick any two angles in the figure below.

a. How many possible angle pairings are there? Explain.

b. Describe the possible relationships between the measures of the angles in each pair. Explain.

c. Describe the likelihood of randomly selecting a pair of congruent angles. Explain your reasoning.

**SOLUTION:**

a. Sample answer: There are 28 possible angle pairings. The first angle can be paired with seven others, then the second angle can be paired with six others since it has already been paired with the first angle. The number of pairings is the sum of the number of angles each subsequent angle can be paired with, \( 7 + 6 + 5 + 4 + 3 + 2 + 1 \) or 28 pairings.

b. Sample answer: Because the two lines being transversed are parallel, there are only two possible relationships between the pairs of angles. Each pair of angles chosen will be either congruent or supplementary.

Congruent pairs: \( \angle 1 \) and \( \angle 3 \), \( \angle 1 \) and \( \angle 5 \), \( \angle 1 \) and \( \angle 7 \), \( \angle 3 \) and \( \angle 5 \), \( \angle 3 \) and \( \angle 7 \), \( \angle 5 \) and \( \angle 7 \), \( \angle 2 \) and \( \angle 4 \), \( \angle 2 \) and \( \angle 6 \), \( \angle 2 \) and \( \angle 8 \), \( \angle 4 \) and \( \angle 6 \), \( \angle 4 \) and \( \angle 8 \), \( \angle 6 \) and \( \angle 8 \)

Supplementary pairs: \( \angle 1 \) and \( \angle 2 \), \( \angle 1 \) and \( \angle 4 \), \( \angle 1 \) and \( \angle 6 \), \( \angle 1 \) and \( \angle 8 \), \( \angle 2 \) and \( \angle 3 \), \( \angle 2 \) and \( \angle 5 \), \( \angle 2 \) and \( \angle 7 \), \( \angle 3 \) and \( \angle 4 \), \( \angle 3 \) and \( \angle 6 \), \( \angle 3 \) and \( \angle 8 \), \( \angle 4 \) and \( \angle 5 \), \( \angle 4 \) and \( \angle 7 \), \( \angle 5 \) and \( \angle 6 \), \( \angle 5 \) and \( \angle 8 \), \( \angle 6 \) and \( \angle 7 \), \( \angle 7 \) and \( \angle 8 \)

c. Sample answer: Twelve of the 28 angle pairs are congruent. So, the likelihood of selecting a pair of congruent angles is \( \frac{12}{28} \) or \( \frac{3}{7} \).

51. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate the relationship between
same-side exterior angles.

a. **GEOMETRY** Draw five pairs of parallel lines, \(m\) and \(n\), \(a\) and \(b\), \(r\) and \(s\), \(j\) and \(k\), and \(x\) and \(y\), cut by a transversal \(t\), and measure the four angles on one side of \(t\).

b. **TABULAR** Record your data in a table.

c. **VERBAL** Make a conjecture about the relationship between the pair of angles formed on the exterior of parallel lines and on the same side of the transversal.

d. **LOGICAL** What type of reasoning did you use to form your conjecture? Explain.

e. **PROOF** Write a proof of your conjecture.

**SOLUTION:**

a. Sample answer for \(m\) and \(n\):

![Diagram of parallel lines and transversal]

b. Sample answer:

<table>
<thead>
<tr>
<th>(m\angle 1)</th>
<th>(m\angle 2)</th>
<th>(m\angle 3)</th>
<th>(m\angle 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>120</td>
<td>60</td>
<td>120</td>
</tr>
<tr>
<td>45</td>
<td>135</td>
<td>45</td>
<td>135</td>
</tr>
<tr>
<td>70</td>
<td>110</td>
<td>70</td>
<td>110</td>
</tr>
<tr>
<td>90</td>
<td>90</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>25</td>
<td>155</td>
<td>25</td>
<td>155</td>
</tr>
</tbody>
</table>

c. Sample answer: In the diagram, \(\angle 1\) and \(\angle 4\) are a pair of exterior angles on the same side of the transversal. The sum of \(m\angle 1\) and \(m\angle 4\) for each row is 60 + 120 = 180, 45 + 135 = 180, 70 + 110 = 180, 90 + 90 = 180, and 25 + 155 = 180. A pair of angles whose sum is 180 are supplementary angles. Therefore, angles on the exterior of a pair of parallel lines located on the same side of the transversal are supplementary.

d. Inductive; a pattern was used to make a conjecture.

e. Given: parallel lines \(m\) and \(n\) cut by transversal \(t\).

Prove: \(\angle 1\) and \(\angle 4\) are supplementary.

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52. **WRITING IN MATH** If line \(a\) is parallel to line \(b\) and \(\angle 1 \equiv \angle 2\), describe the relationship between lines \(b\) and \(c\). Explain your reasoning.

**SOLUTION:**

Lines \(b\) and \(c\) are perpendicular. Since \(\angle 1 \) and \(\angle 2\) form a linear pair, \(m\angle 1 + m\angle 2 = 180\). \(\angle 1 \equiv \angle 2\), so \(m\angle 1 = m\angle 2\). Substituting, \(m\angle 1 + m\angle 1 = 180\), so \(m\angle 1 = 90\) and \(m\angle 2 = 90\). So, lines \(a\) and \(c\) are perpendicular. By Theorem 3.4, since transversal \(c\) is perpendicular to line \(a\) and lines \(a\) and \(b\) are parallel, then line \(c\) is perpendicular to line \(b\).

53. **WRITING IN MATH** Compare and contrast the Alternate Interior Angles Theorem and the Consecutive Interior Angles Theorem.

**SOLUTION:**

In both theorems, a pair of angles is formed when two parallel lines are cut by a transversal. However, in the Alternate Interior Angles Theorem, each pair of alternate interior angles that is formed are congruent, whereas in the Consecutive Interior Angles Theorem, each pair of angles formed is supplementary.
54. **OPEN ENDED** Draw a pair of parallel lines cut by a transversal and measure the two exterior angles on the same side of the transversal. Include the measures on your drawing. Based on the pattern you have seen for naming other pairs of angles, what do you think the name of the pair you measured would be?

**SOLUTION:**

Consecutive Exterior Angles or Same-Side Exterior Angles

55. **CHALLENGE** Find $x$ and $y$.

**SOLUTION:**

To find $x$ and $y$, we will write two equations and solve the system. In the figure, we are given a pair of consecutive interior angles $[x^\circ \text{ and } y^\circ]$, alternate interior angles $(y^\circ \text{ and } \left(8y - 15\right)^\circ)$, and supplementary angles $[x^\circ \text{ and } \left(8y - 15\right)^\circ]$. Use the supplementary angles and consecutive interior angles since they are both equal to 180.

Supplementary angles equation: $x + 8y - 15 = 180$

Consecutive Interior angles equation: $x + y^2 = 180$

Name the equation.

$\begin{align*}
&x + 8y - 15 = 180 \quad \text{Equation 1} \\
&x + y^2 = 180 \quad \text{Equation 2}
\end{align*}$

Subtract equations

$\begin{align*}
y^2 + 8y - 15 &= 0 \\
\therefore (y - 3)(y - 5) &= 0 \\
\therefore y &= 3 \text{ or } 5
\end{align*}$

Substitute $y = 3$ in Equation 2.

$x + 3^2 = 180 \quad \text{Equation 2}$

$x + 9 = 180$

$x = 171$

Substitute $y = 5$ in Equation 2.

$x + 5^2 = 180 \quad \text{Equation 2}$

$x + 25 = 180$

$x = 155$

Thus, $x = 171$ or $x = 155$. 
56. **REASONING** Determine the minimum number of angle measures you would have to know to find the measures of all the angles formed by two parallel lines cut by a transversal. Explain.

**SOLUTION:**
One is the minimum number of angle measures you would have to know to find the measures of all the angles formed by two parallel lines cut by a transversal.
Once the measure of one angle is known, the rest of the angles are either congruent or supplementary to the given angle.

57. In the diagram, \( m\parallel n \). If \( m\angle 1 = 4x - 6 \) and \( m\angle 7 = 2x + 40 \), what is \( m\angle 4 \)?

\[\begin{align*}
4x - 6 &= 2x + 40 \\
4x &= 2x + 46 \\
x &= 23 \\
4(23) - 6 &= 86 \\
86 + m\angle 4 &= 180 \\
m\angle 4 &= 94
\end{align*}\]

A 23  
B 86  
C 94  
D 157

**SOLUTION:**
If \( m\angle 1 = 4x - 6 \) and \( m\angle 7 = 2x + 40 \), and the lines are parallel, then \( 4x - 6 = 2x + 40 \). Solve the equation, then find the measure of angle 4 using supplementary angles.

58. In the diagram, \( m\angle 8 = 11y + 7 \) and \( m\angle 3 = y + 17 \). What is \( m\angle 2 \)?

**A** 13  
**B** 30  
**C** 150  
**D** 167

**SOLUTION:**
\( \angle 8 \) and \( \angle 2 \) are alternate exterior angles. Thus \( m\angle 8 = m\angle 2 \).
Then \( \angle 2 \) and \( \angle 3 \) form a linear pair. Use the definition of linear pair to find \( y \) and \( m\angle 2 \).

\[
\begin{align*}
(11y + 7) + (y + 17) &= 180 \\
12y + 24 &= 180 \\
12y + 24 - 24 &= 180 - 24 \\
12y &= 156 \\
\frac{12y}{12} &= \frac{156}{12} \\
y &= 13 \\
m\angle 2 &= 13(13) + 7x 150 \\
\text{Reduce} 2
\end{align*}\]

The correct choice is C.

59. Which angles are consecutive angles?

**A** \( \angle 1 \) and \( \angle 5 \)  
**B** \( \angle 3 \) and \( \angle 4 \)  
**C** \( \angle 4 \) and \( \angle 6 \)  
**D** \( \angle 4 \) and \( \angle 5 \)

**SOLUTION:**
Consecutive angles are non-overlapping and they share a ray. Of the answer choices only \( \angle 3 \) and \( \angle 4 \) meet those requirements.
60. Which of the following pairs of angles would not necessarily be supplementary?

F Angles that would form a straight line
G Corresponding angles
H Any pair of angles in a rectangle
J Consecutive interior angles

**SOLUTION:**
F Angles that would form a straight line are always supplementary, since angles on a straight line are 180°.
H Any pair of angles in a rectangle is true since all angles are 90° and any pair will always be supplementary.
J Consecutive interior angles are always supplementary.
Thus, the correct choice is G. Corresponding angles are not necessarily supplementary.

61. In the figure which pairs of angles are corresponding angles? Select all of the pairs in the figure.

A \( \angle 1 \) and \( \angle 5 \)
B \( \angle 2 \) and \( \angle 8 \)
C \( \angle 3 \) and \( \angle 7 \)
D \( \angle 4 \) and \( \angle 8 \)
E \( \angle 5 \) and \( \angle 7 \)
F \( \angle 6 \) and \( \angle 8 \)

**SOLUTION:**
Corresponding angles are in the same position but at different intersections of lines, so only E \( \angle 5 \) and \( \angle 7 \), and F \( \angle 6 \) and \( \angle 8 \) are corresponding angles.

62. **MULTI-STEP** Mitchell is designing the parking lot for a new shopping center. The figure below shows a plan for several of the new spaces in the parking lot. In the figure, lines \( m \), \( n \), and \( p \) are parallel to each other.

![Diagram of parallel lines and transversals]

a. Classify the relationship between angle 4 and angle 9.

b. Suppose Mitchell decides that \( m \angle 9 = 150 \). Find \( m \angle 3 \).

c. Is it possible for \( \angle 4 \cong \angle 10 \)? If so, what must be true about the angles? If not, why not? Explain.

d. Suppose Mitchell decides that \( m \angle 2 = 90 \). In this case what could he conclude about lines \( t \) and \( p \)? Explain.

**SOLUTION:**
a. Angle 4 and angle 9 are alternate interior angles.

b. If \( m \angle 9 = 150 \) then \( m \angle 3 + 150 = 180 \), because they are same side interior angles and are supplementary. So \( m \angle 3 = 30 \)

c. \( \angle 4 \cong \angle 10 \) only if line \( t \) is perpendicular to lines \( m \) and \( p \). Then the angles would be right angles.

d. If \( m \angle 2 = 90 \), then line \( t \) is perpendicular to line \( p \), because right angles measure 90 degrees and perpendicular lines meet at right angles.